

# Department of Automotive Technologies – Vehicle Mechanics Fundamentals

**Gábor Sipos**



Lecture 2

# Base information

Week nr.	Date		Lecture (Wednesday)		Lab (Wednesday)
1	12th Feb	1	General information, Tyre, Driving force	1	Lab
2	19th Feb	2	Longitudinal and lateral behaviour		
3	26th Feb	3	Concepts and over/understeer	2	Lab
4	5th Mar	4	Weight transfer		
5	12th Mar	5	Bicycle model	3	Lab
6	19th Mar	T1	Midterm exam I.		
7	26th Mar	6	Braking and brakes	T1 R	Exam 1 - replacement
8	2nd Apr	-	Systems of the vehicle		
9	9th Apr	7	Break		Break
10	16th Apr	8	Quarter vehicle model		
-	23th Apr		Break		Break
11	30th Apr	T2	Systems of vehicle II. ONLINE	4	Lab
12	7th May	9	Tyre management		
13	14th May	10	Midterm exam II.	11	Racecar engineering
14	21st May	T2 R	Exam 2 - replacement		

Schedule  
change!

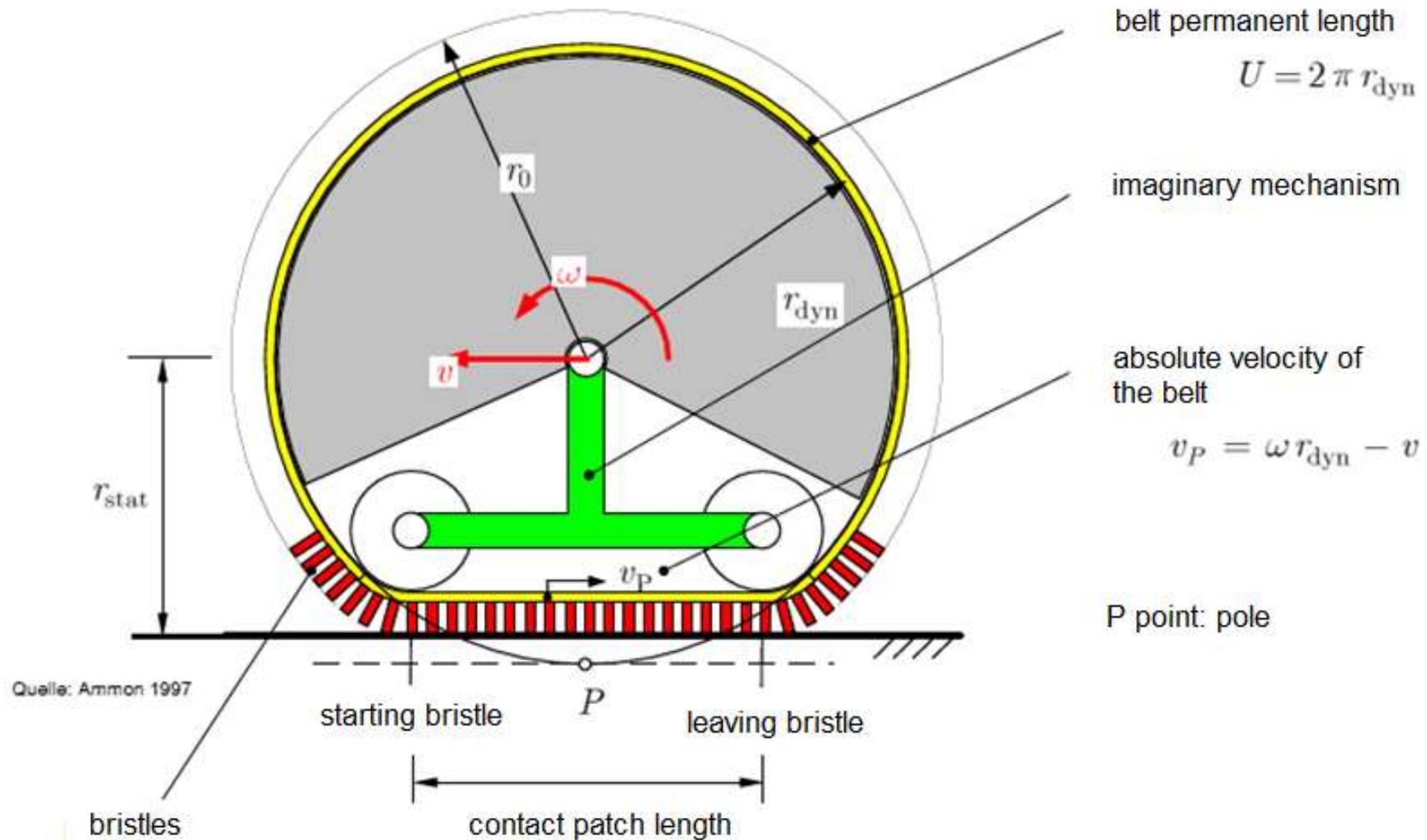
## Longitudinal and lateral behaviour

- Brush model
- Degressivity of tyre
- Friction coefficient useage
- Lateral dynamics – slip angle
- Tyre behaviours
- Kamm-circle
- Tyres forces and torques
- Cornering stiffness, self-aligning torque

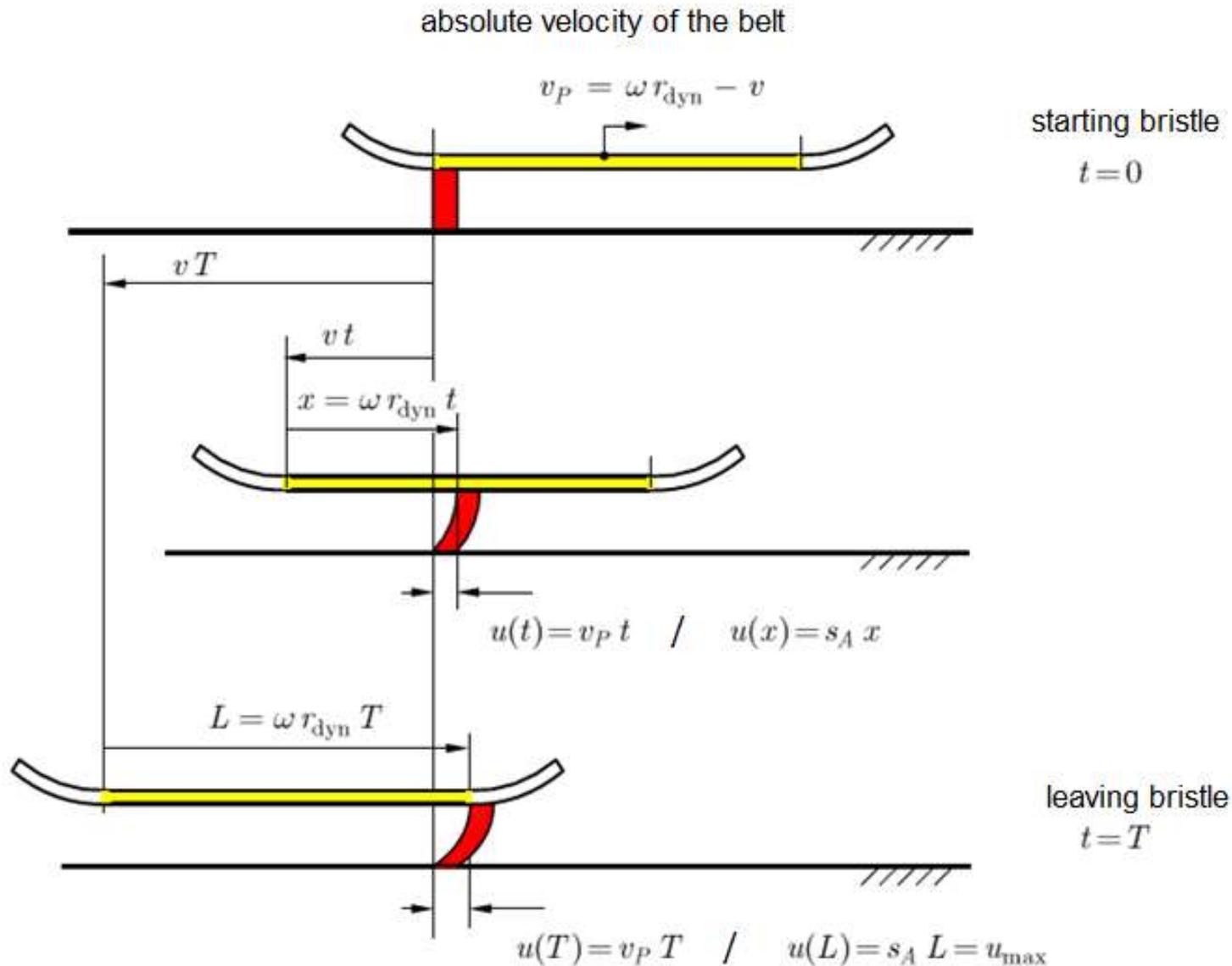
## Note

To prepare for exam/tests presentations will be uploaded to Moodle, but the some of the slides aren't – therefore to have the full picture lectures shall be attended.

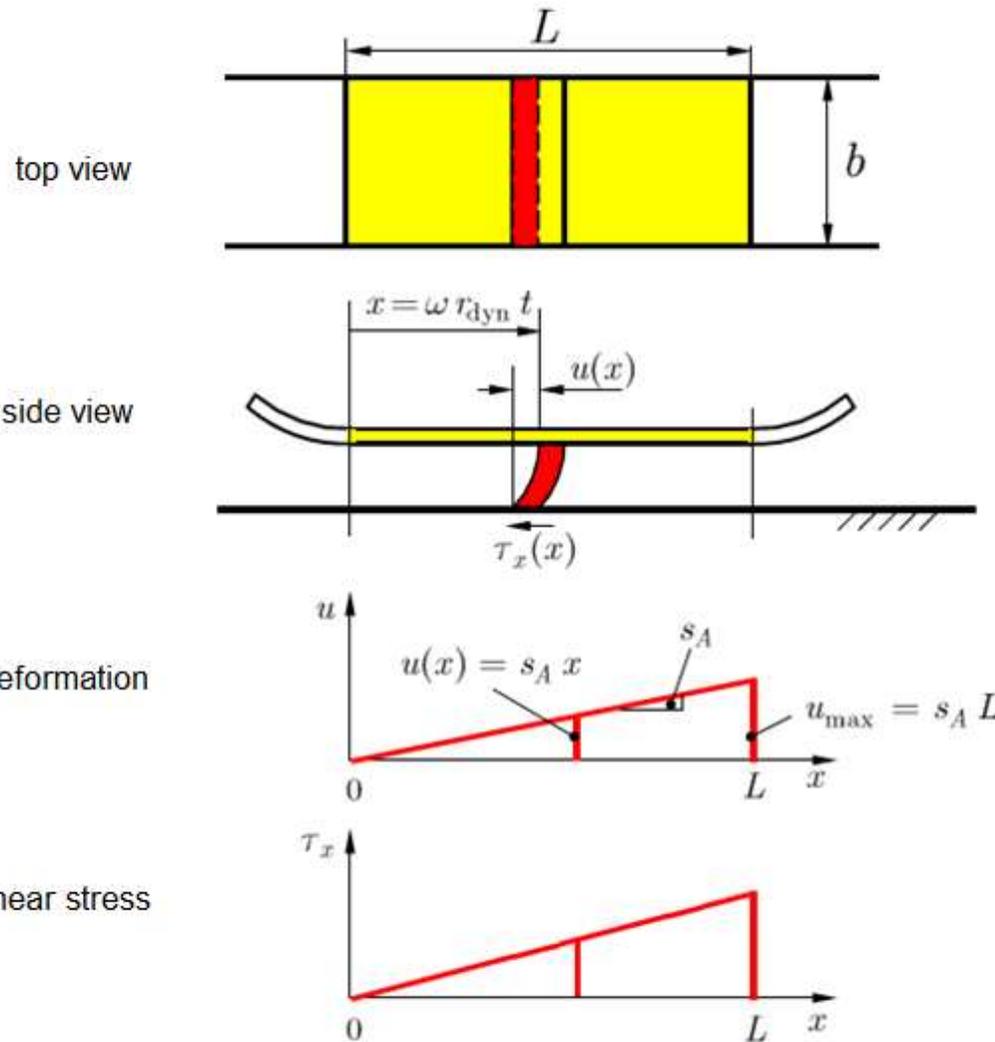
# Brush model



# Brush model



# Brush model



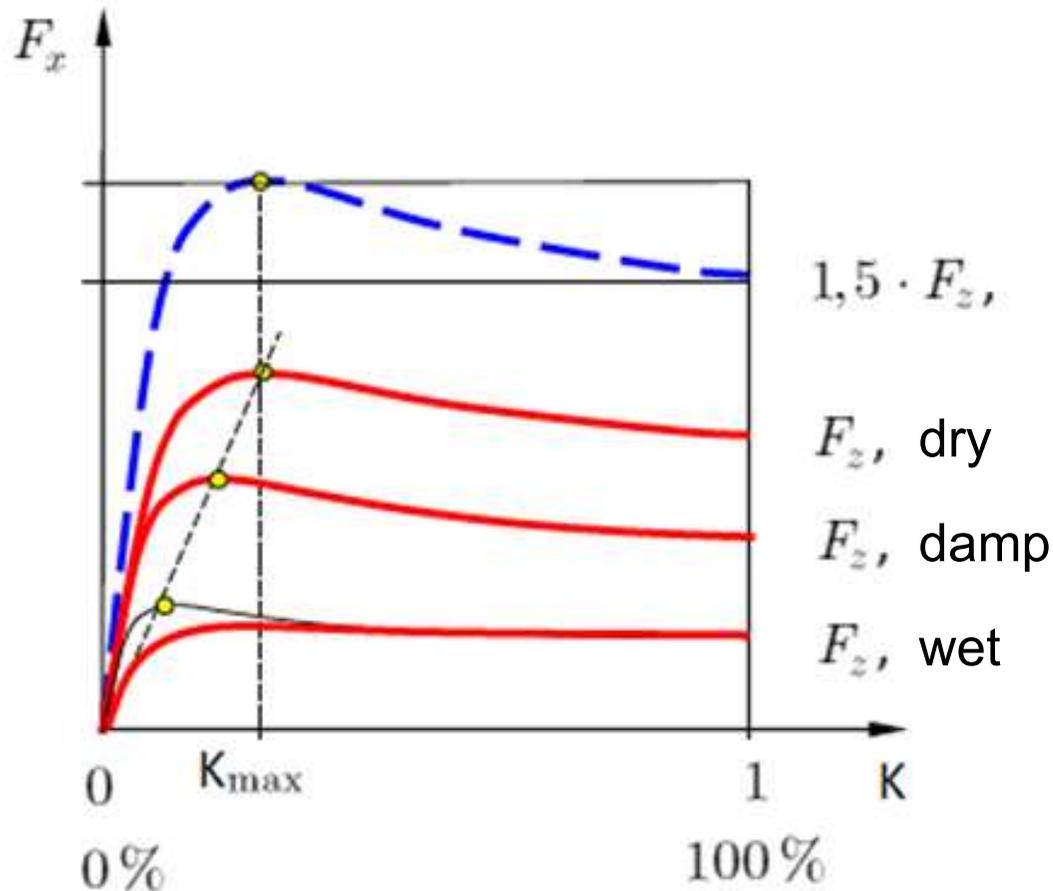
assumption:

contacting on full length of CP

Longitudinal force by bristle

$$F_x = \int_A \tau_x(x) dA$$

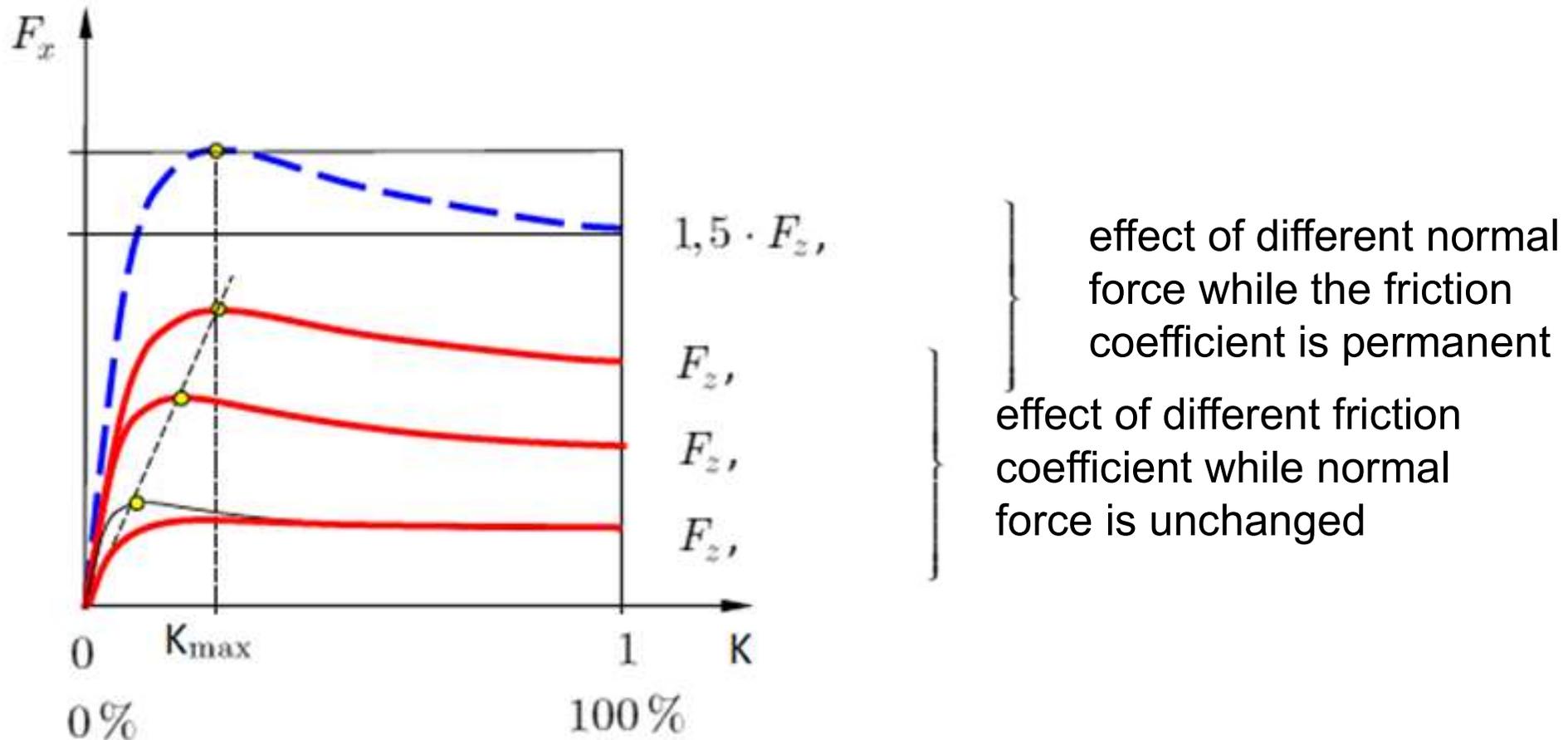
# Effect of factors to longitudinal force - SR



} effect of different normal force while the friction coefficient is permanent

} effect of different friction coefficient while normal force is unchanged

# Effect of factors to longitudinal force - SR



**Question:** if the more weight effect is as advantageous as it seems here, why don't we start building tanks?

# Grip or power limited?

## Newton's second law

Power limited

$$F_x = m \cdot a$$



To have the best  
acceleration

Keep low as possible

Grip limited

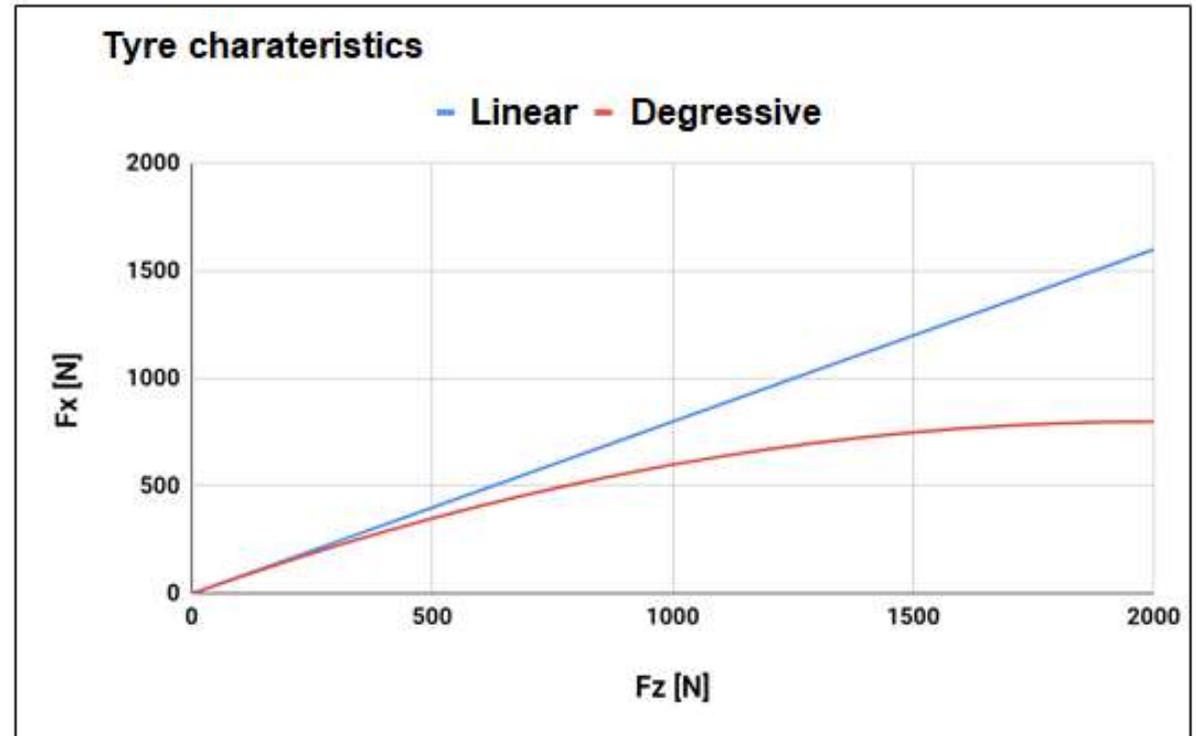
$$F_x = m \cdot a$$

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x}{m} = \frac{m \cdot g \cdot \mu_x}{m} = g \cdot \mu_x$$

# Buliding tanks or not?

Linear characteristics:

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x}{m} = \frac{m \cdot g \cdot \mu_x}{m} = g \cdot \mu_x$$

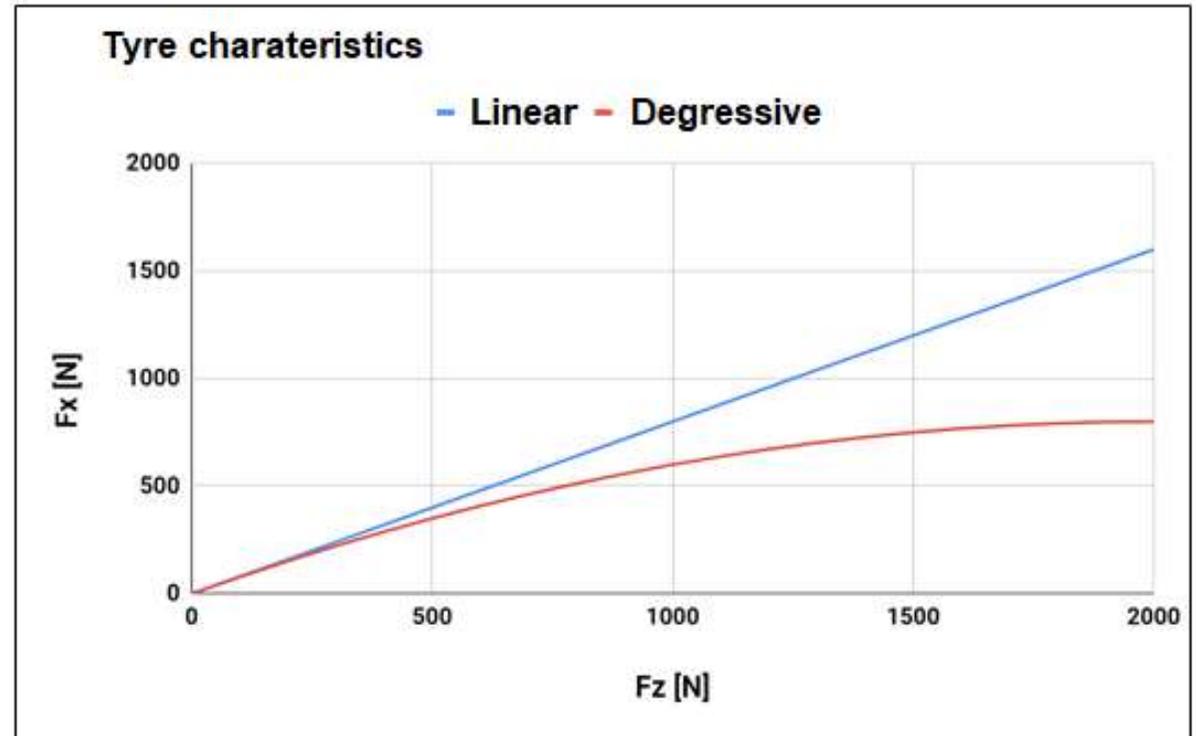


Linear characteristics:

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x}{m} = \frac{m \cdot g \cdot \mu_x}{m} = g \cdot \mu_x$$

Degressive characteristics:

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x - F_z^2 \cdot \mu_{x,2}}{m}$$

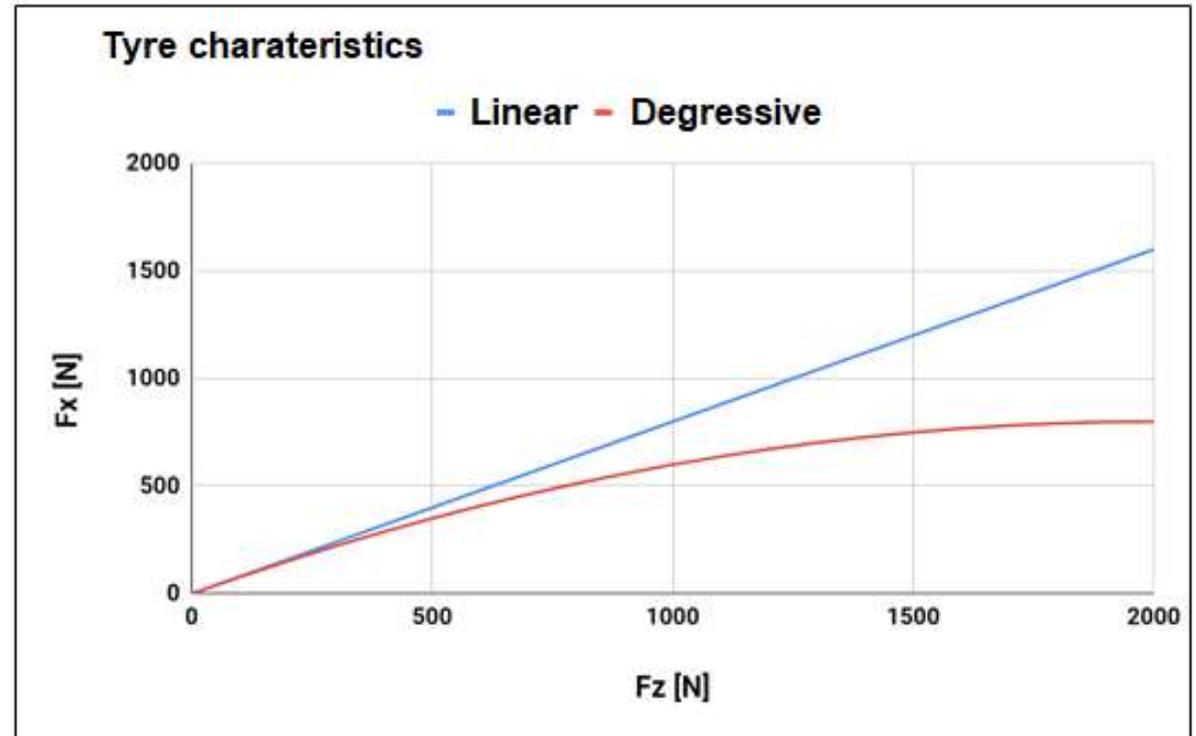


# Buliding tanks or not?

Tyre characteristics is degressive!

Linear characteristics:

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x}{m} = \frac{m \cdot g \cdot \mu_x}{m} = g \cdot \mu_x$$

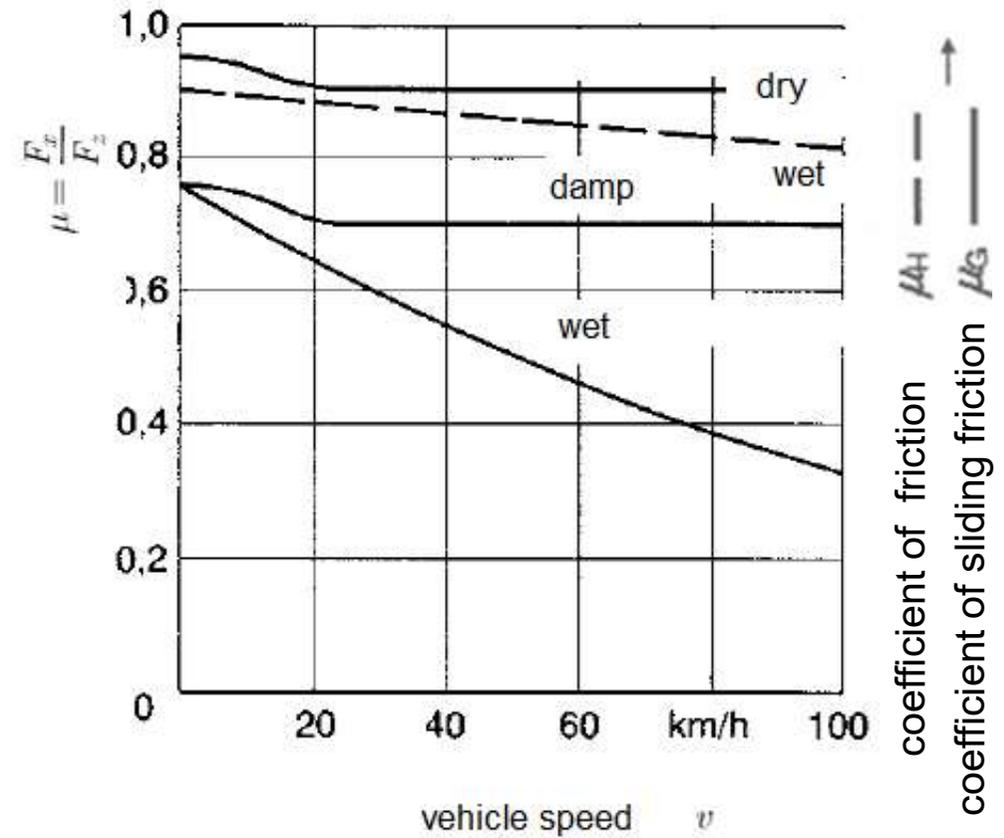
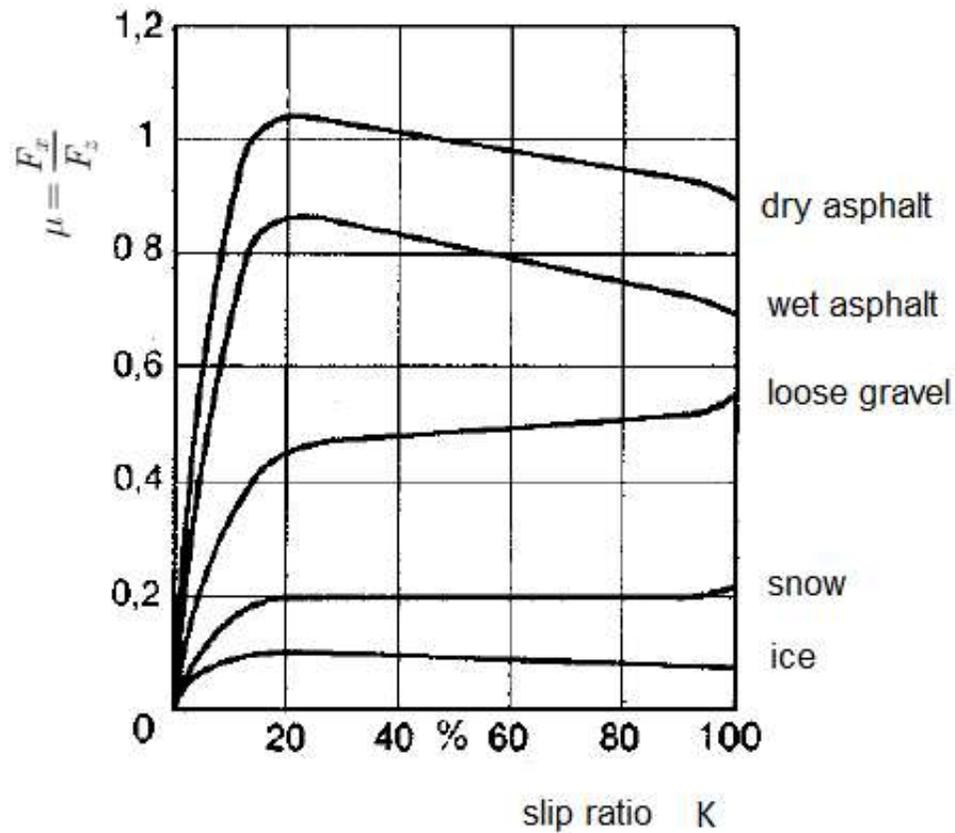


Degressive characteristics:

$$a = \frac{F_x}{m} = \frac{F_z \cdot \mu_x - F_z^2 \cdot \mu_{x,2}}{m} = \frac{m \cdot g \cdot \mu_x - m^2 \cdot g^2 \cdot \mu_{x,2}}{m} = g \cdot \mu_x - m \cdot g^2 \cdot \mu_{x,2}$$

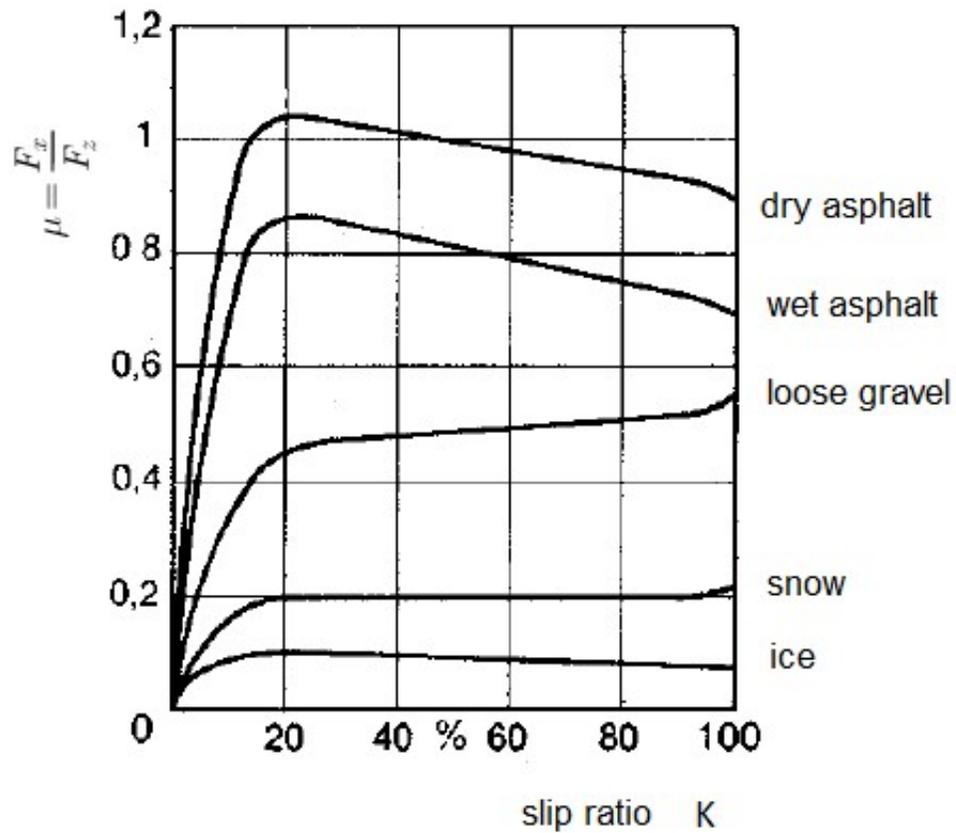
The more vehicle weight results **less** max. acceleration!

# Friction coefficient $\mu_x$



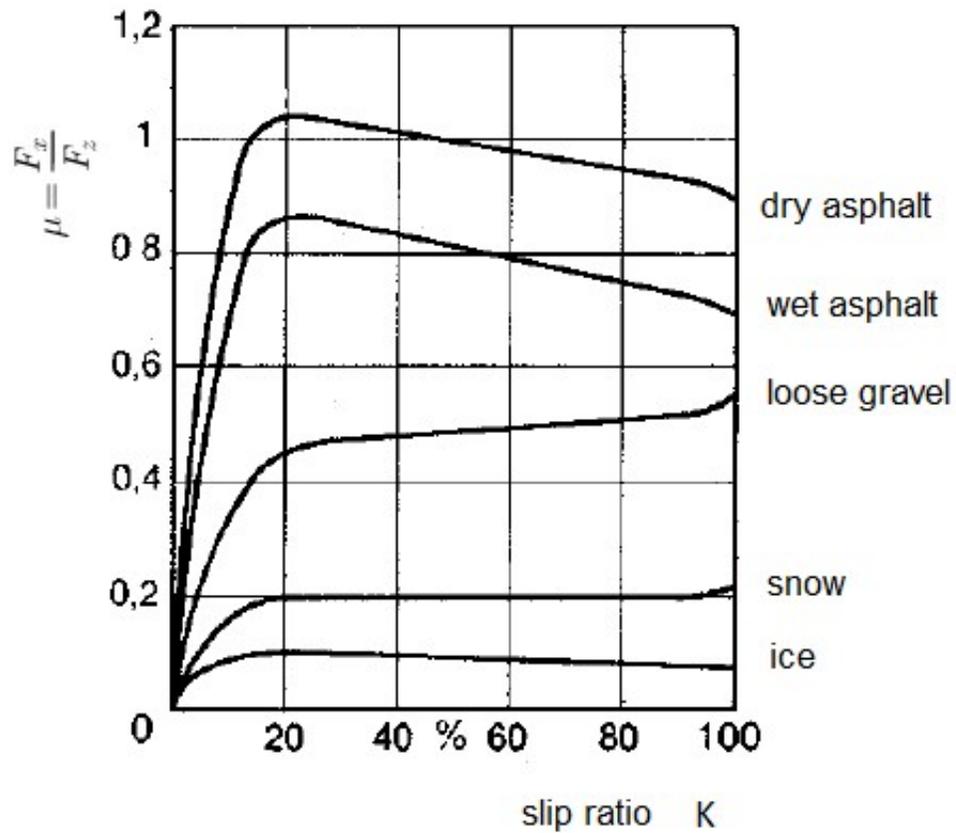
Source: Zomotor Ádám: Gépjármű menetdinamika, 2003

# Friction coefficient $\mu_x$



value of  $\mu_x$  in F1?

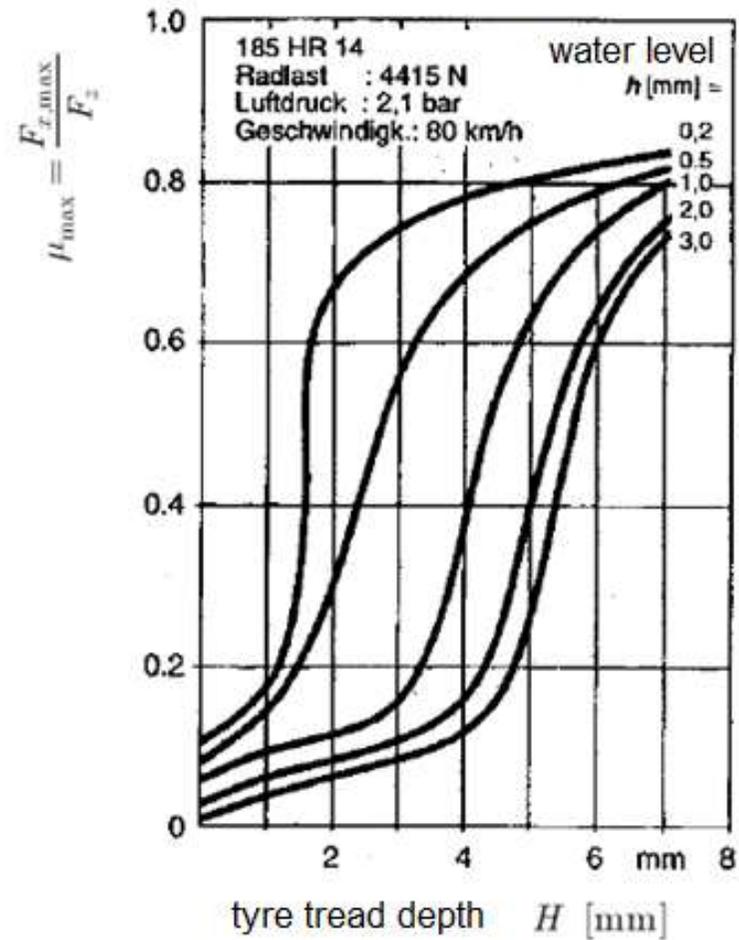
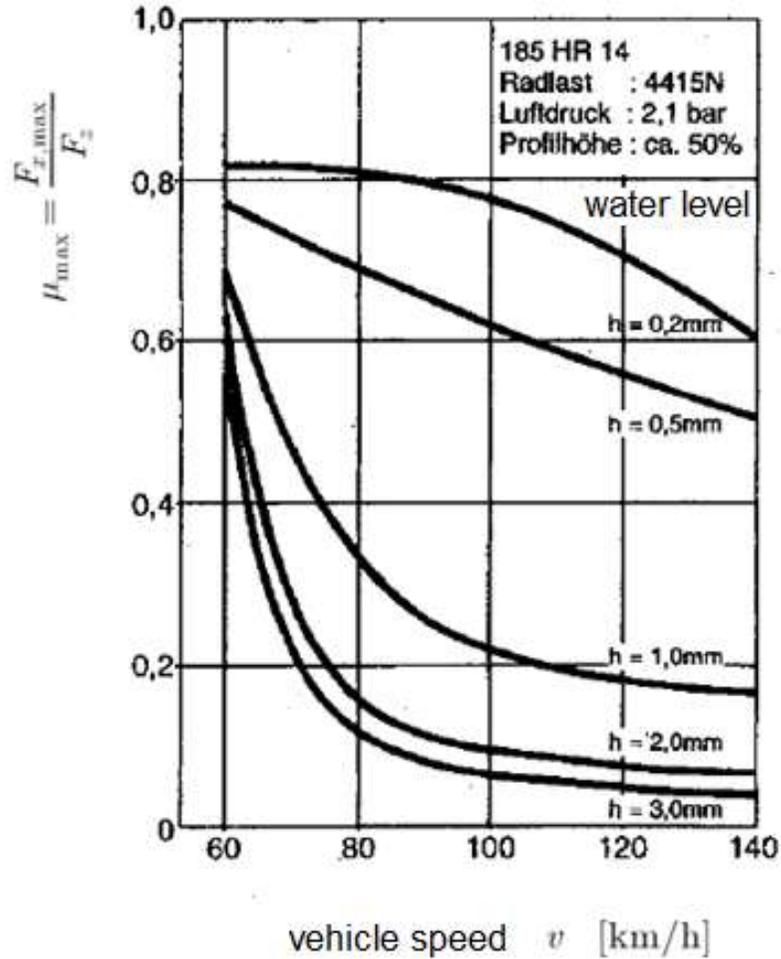
# Friction coefficient $\mu_x$



value of  $\mu_x$  in F1?

1.5-1.7

# Friction coefficient $\mu_x$



Quelle: Wallentowitz 1997

# TYRE SAFETY MONTH

Tyre wear affects vehicle performance and safety. The deeper the groove, the more grip and traction your tyre will have.

Make sure you regularly check the tread depth of your tyres and replace them when they are worn.



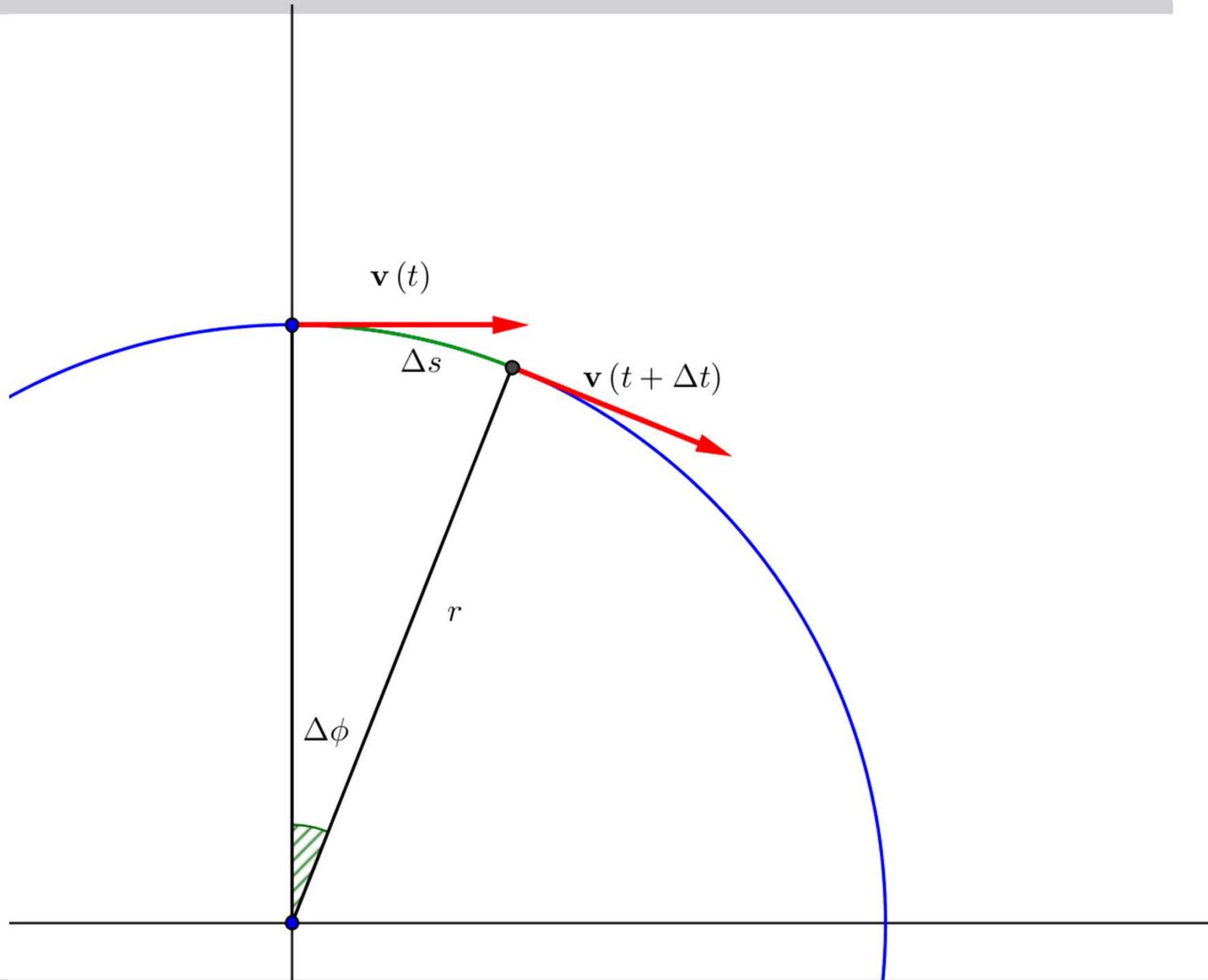
**1.6mm**  
legal minimum tread depth



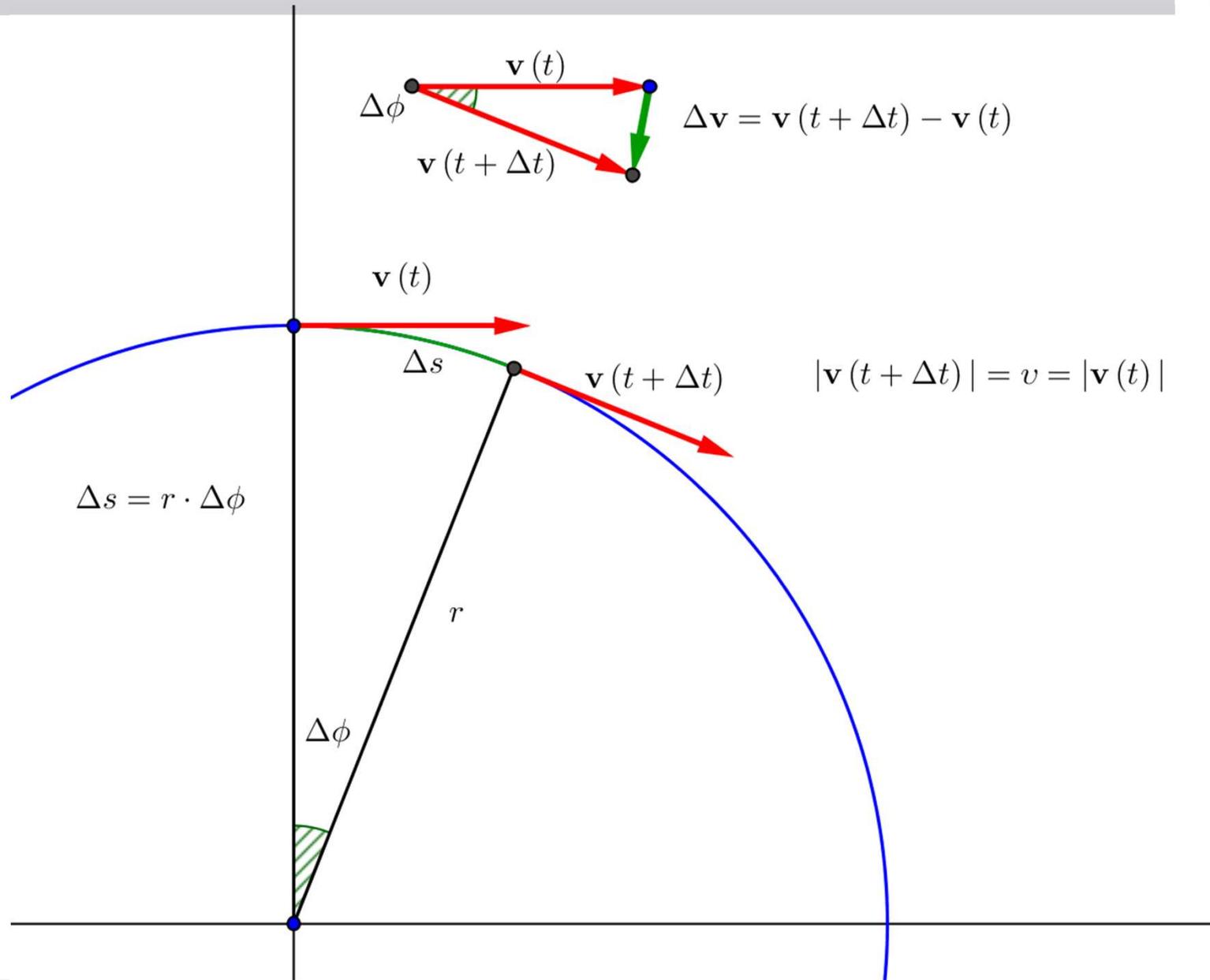
# Day in the office – for tires



# Lateral dynamics – steady state condition

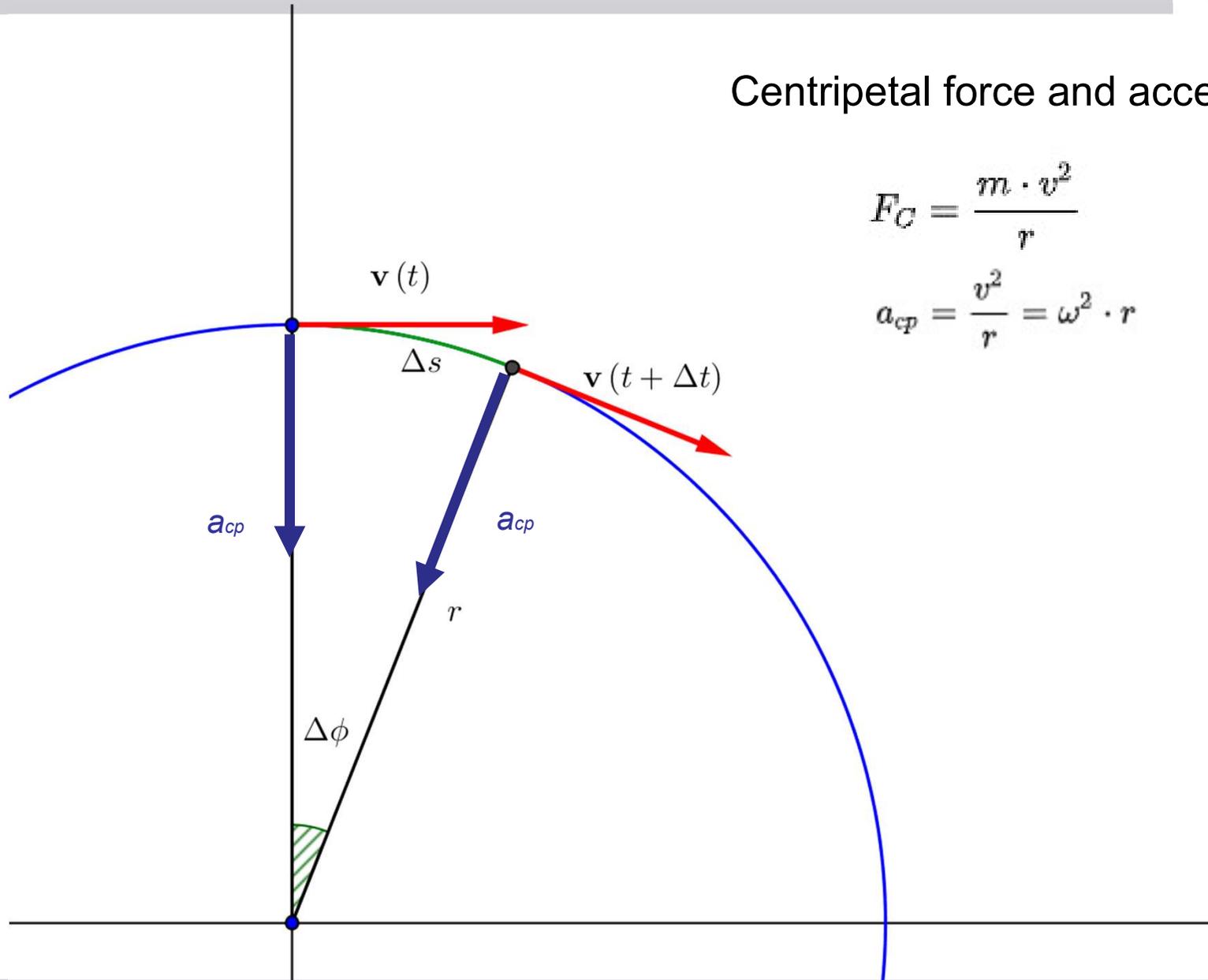


# Lateral dynamics - steady state condition



# Lateral dynamics - steady state condition

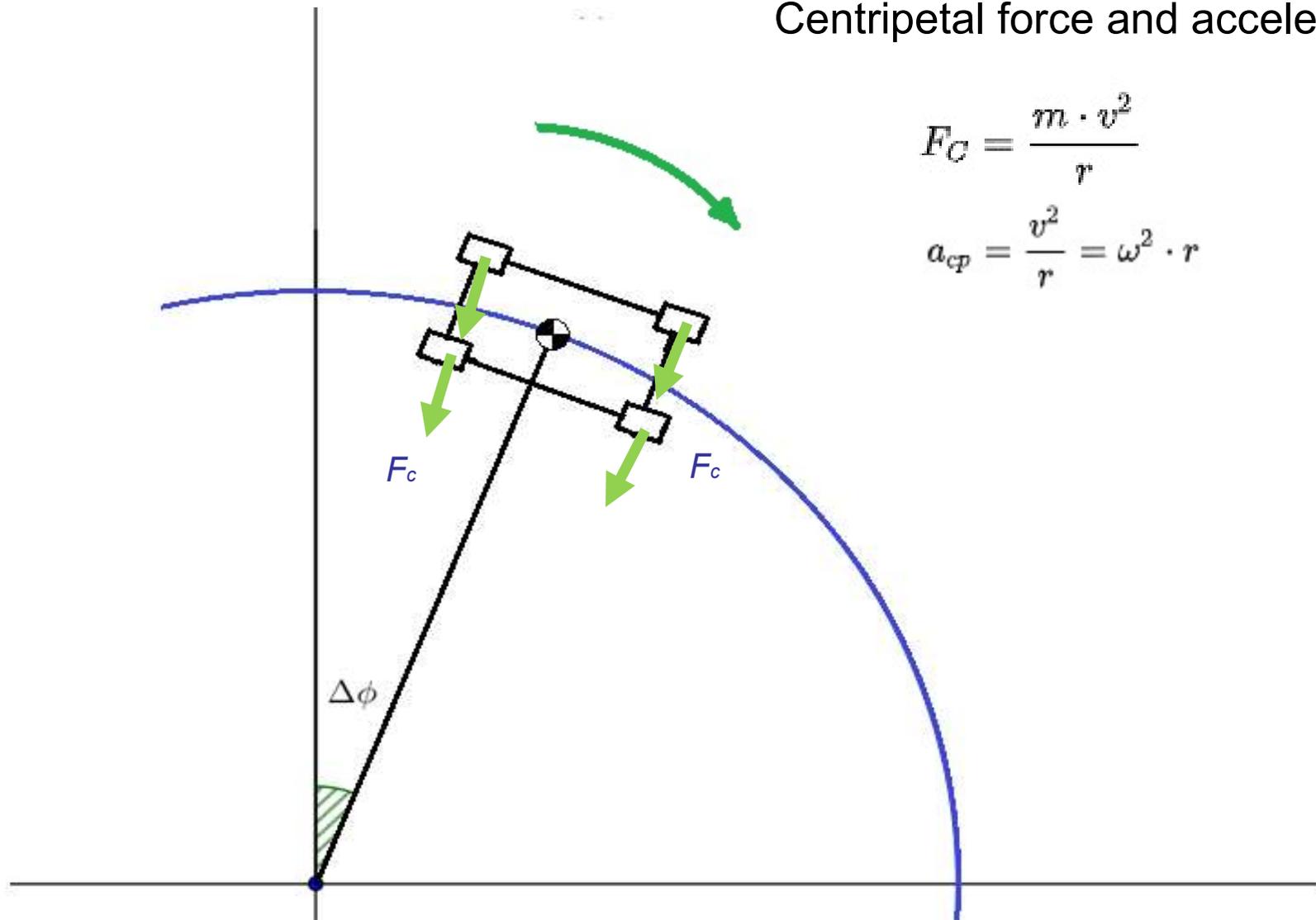
Centripetal force and acceleration



$$F_C = \frac{m \cdot v^2}{r}$$

$$a_{cp} = \frac{v^2}{r} = \omega^2 \cdot r$$

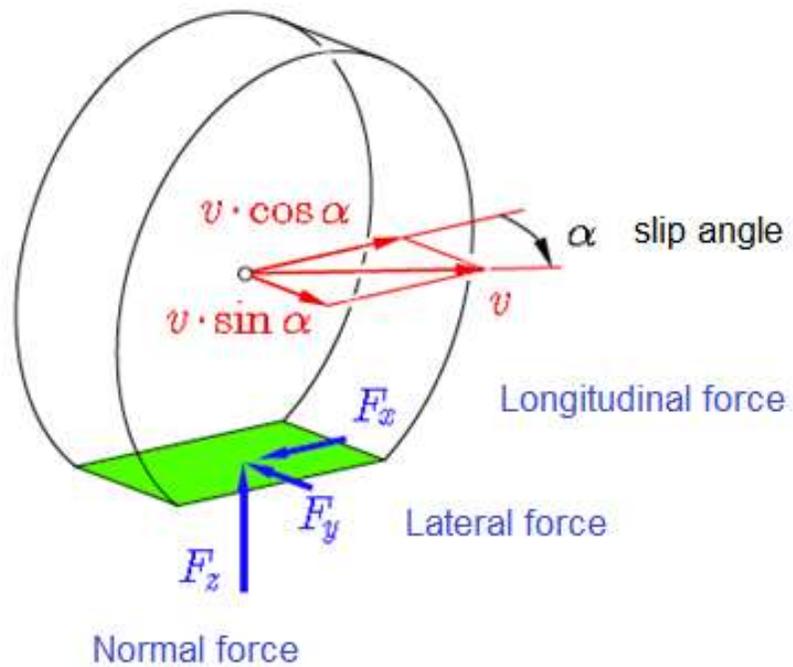
Centripetal force and acceleration



$$F_C = \frac{m \cdot v^2}{r}$$

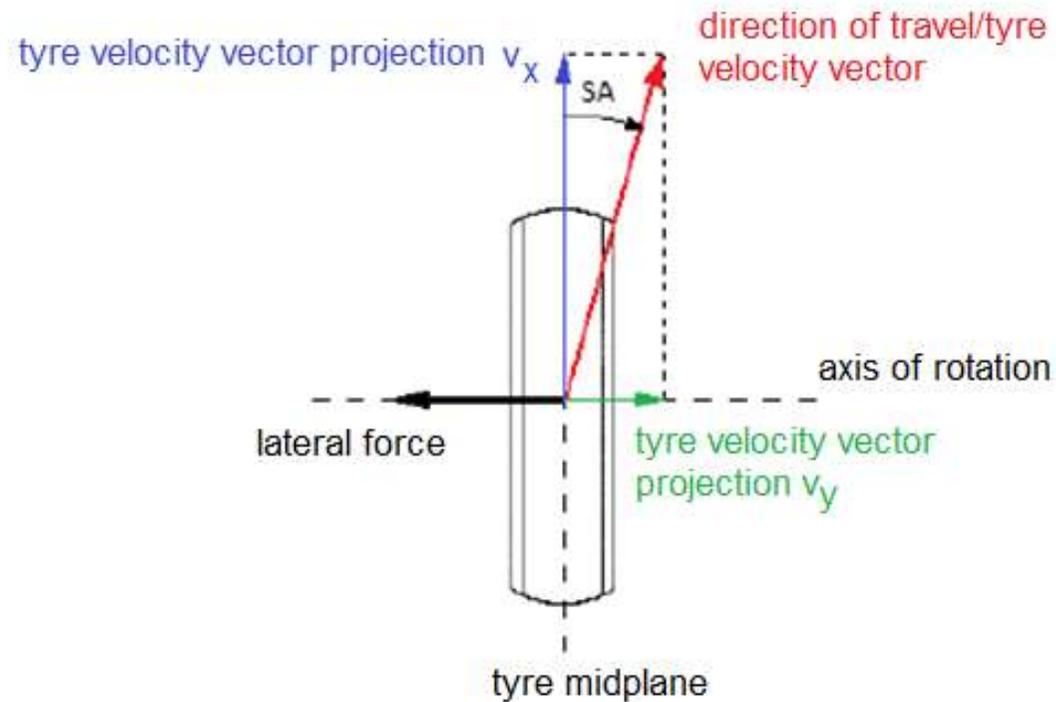
$$a_{cp} = \frac{v^2}{r} = \omega^2 \cdot r$$

# Tyre forces, slip angle



$\alpha = SA = \text{slip angle}$

# Slip angle definition



Slip angle definition:

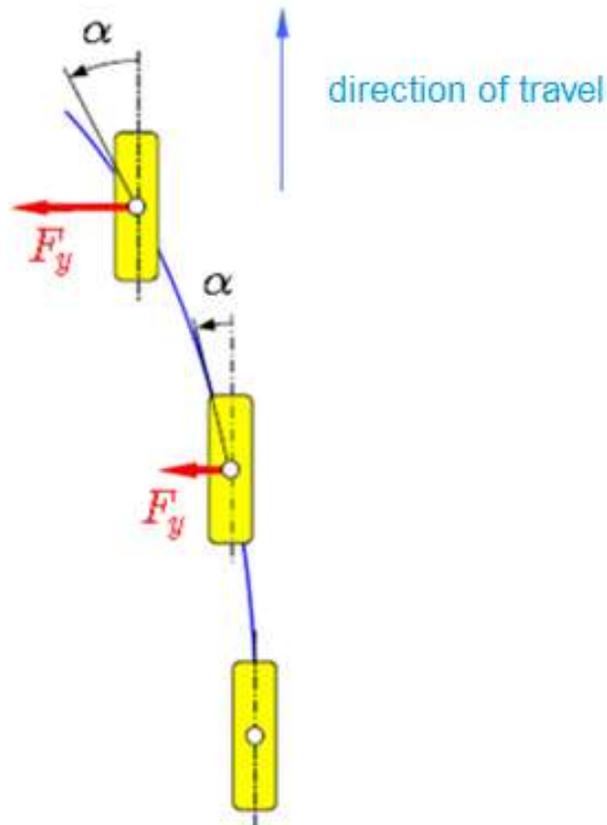
$$SA = \arctan\left(\frac{v_y}{v_x}\right)$$

The angle between actual tyre velocity vector and the tyre midplane is the slip angle.

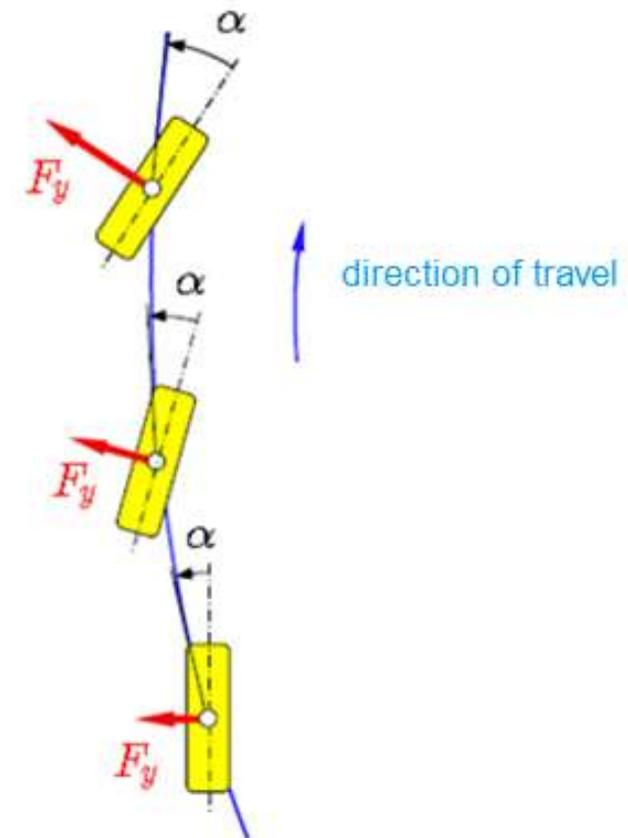
# Slip angle video



# Slip angle and lateral force

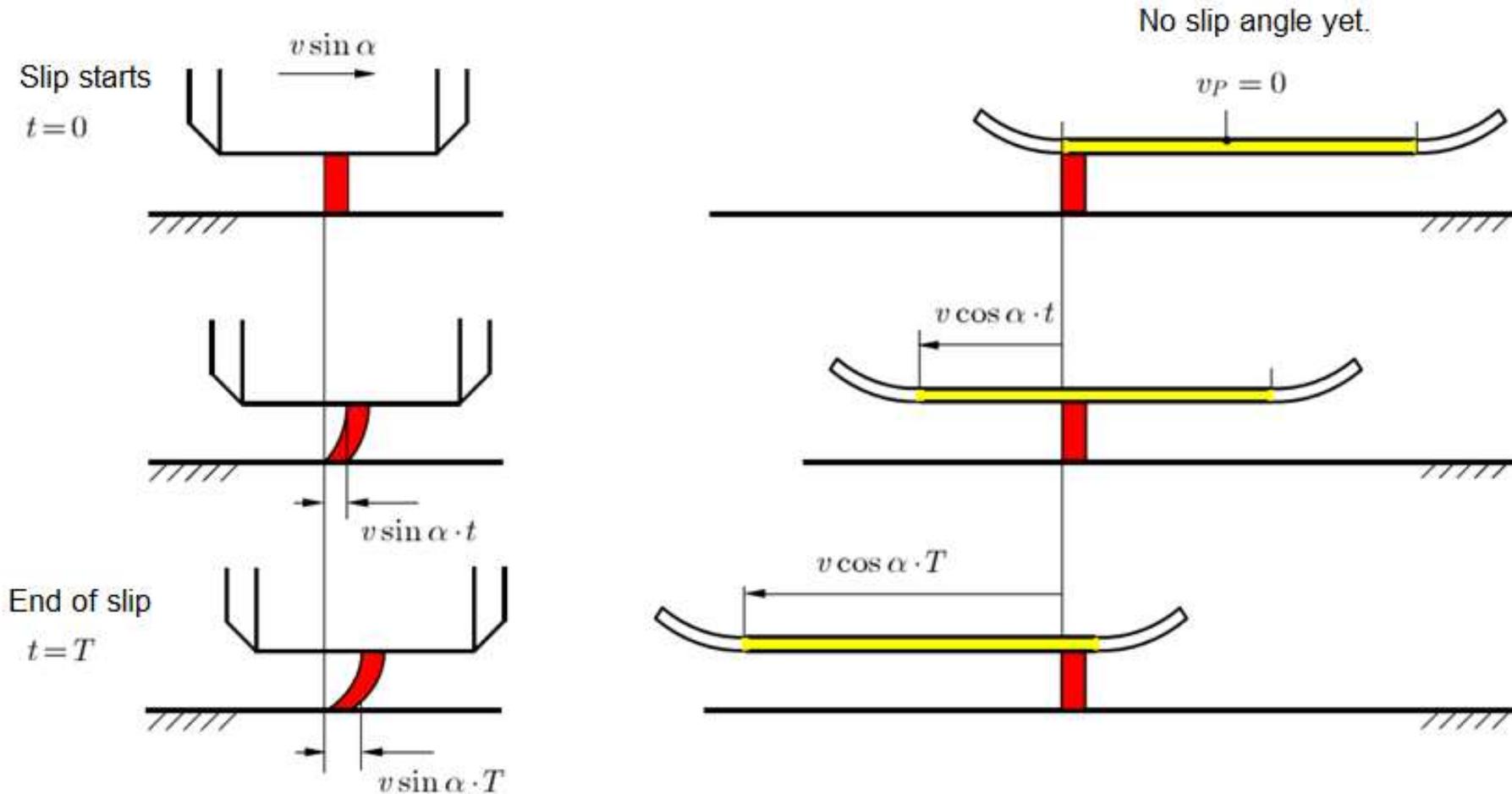


by increasing lateral force  $\rightarrow$  SA will be increasing



by increasing SA  $\rightarrow$  lateral force will be increasing

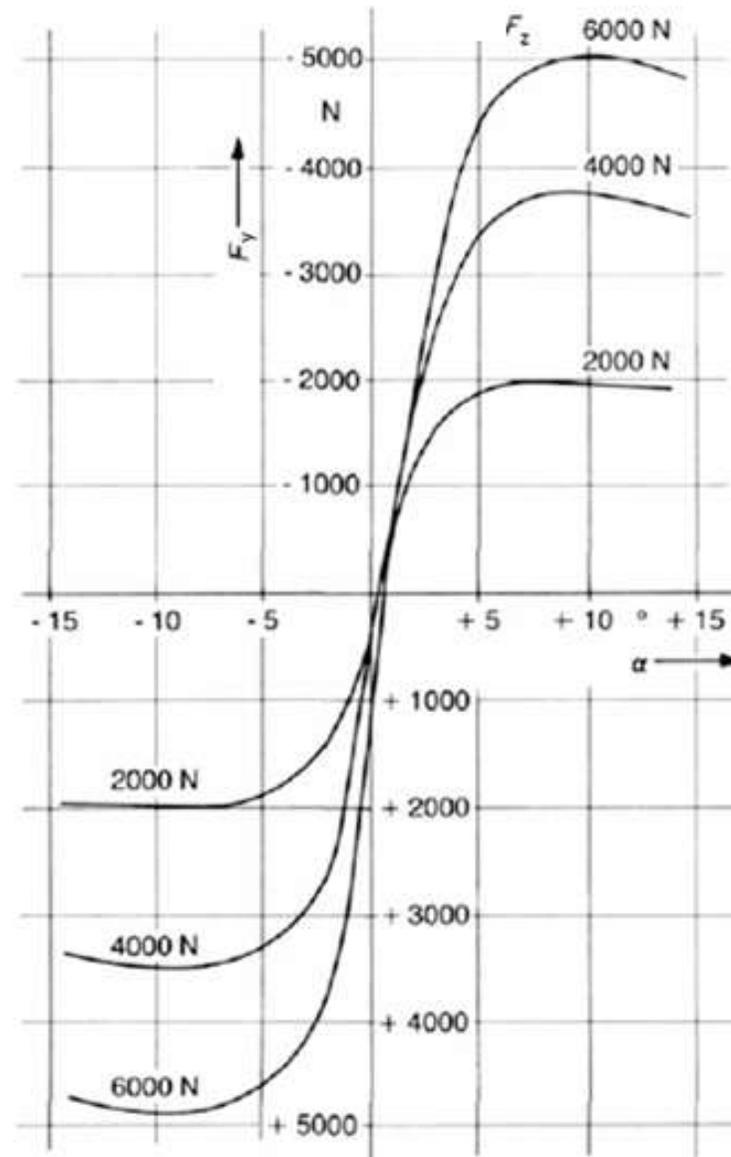
# Brush model for lateral behaviour



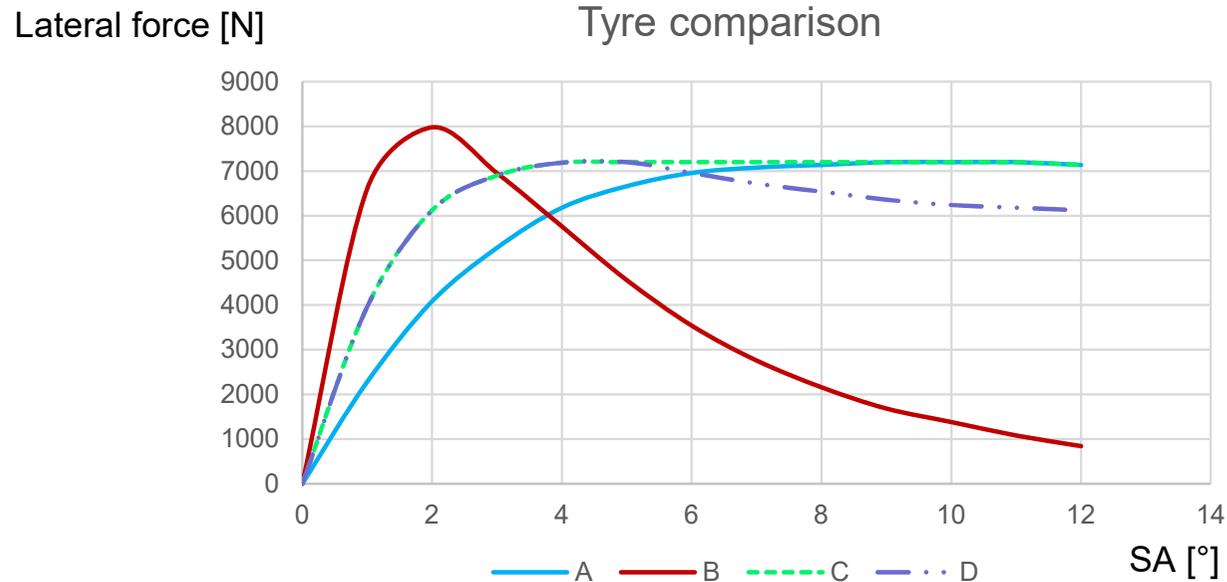
The deformation of one bristle.

Source: C. Woernle

# Correlation of a tyre lateral force – SA



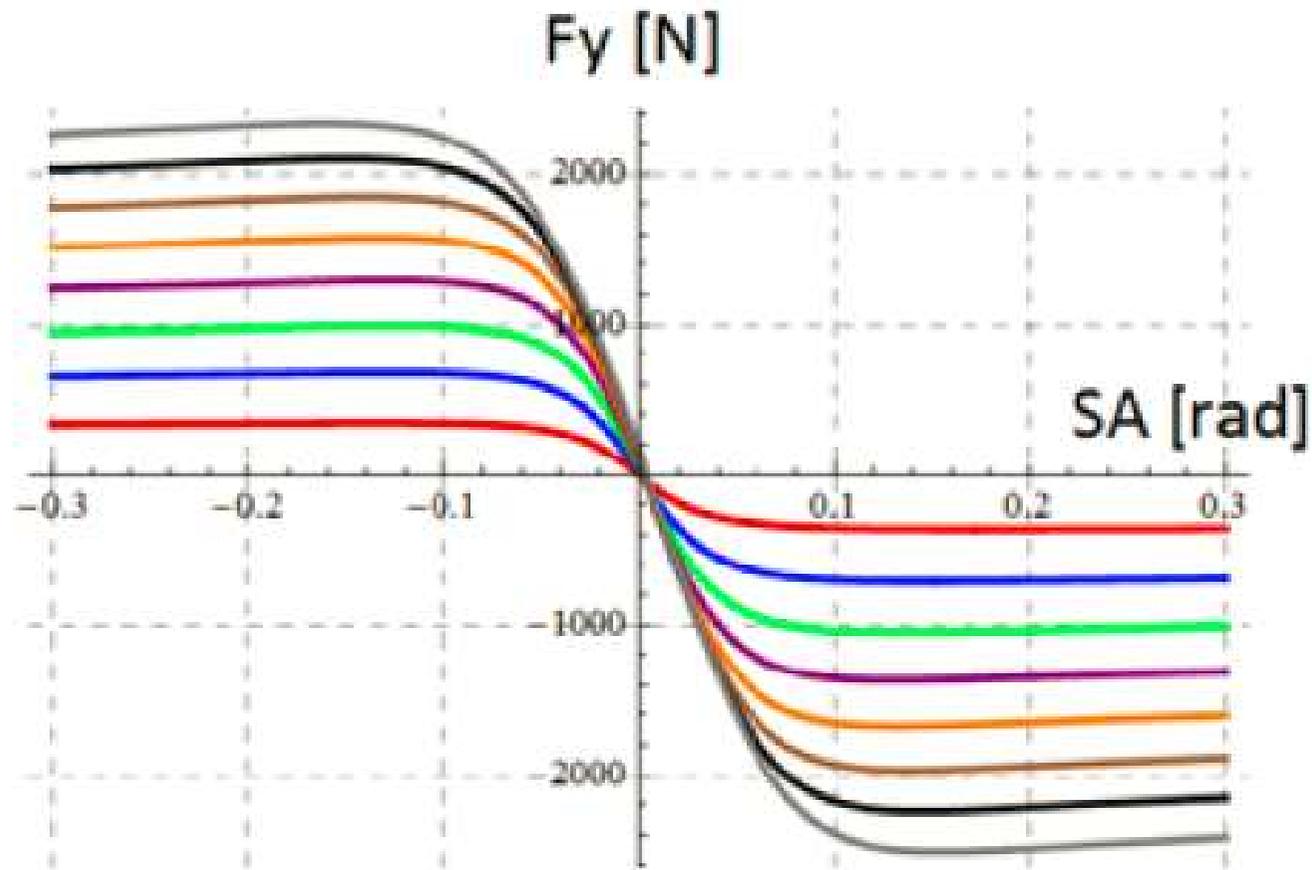
# Different tyre characteristics – $F_y$ - SA



	<b>A</b>	<b>B</b> <i>In theory the fastest</i>	<b>C</b>	<b>D</b>
reaction	slow	required good driver	difficult to drive	quick
feedback	good	best	minimum	less
energy loss	high	minimum	highest	less

# Lateral force

What could the colours mean?



—  $F_z=200$  —  $F_z=400$  —  $F_z=600$  —  $F_z=800$  —  $F_z=1000$  —  $F_z=1200$  —  $F_z=1400$  —  $F_z=1600$

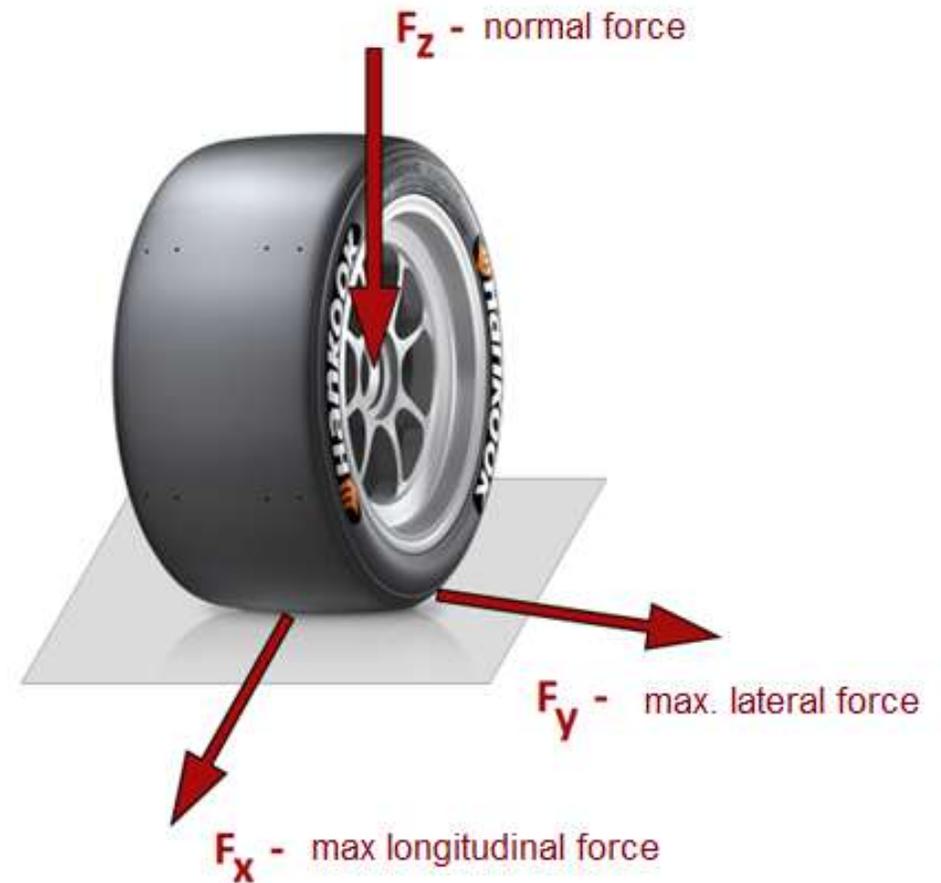
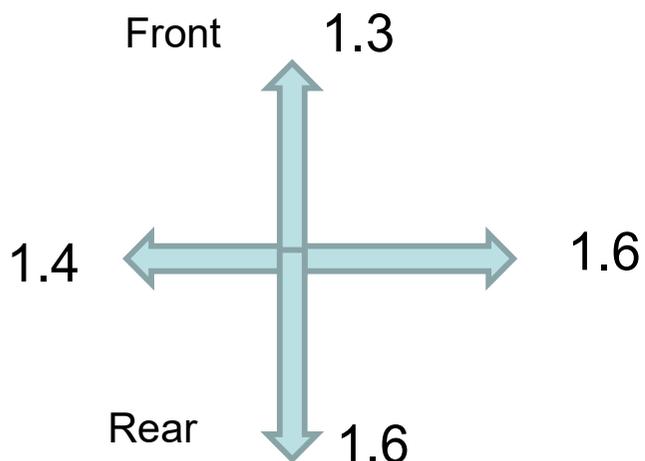
# Friction coefficient $\mu_y$

## Friction coefficient definitions

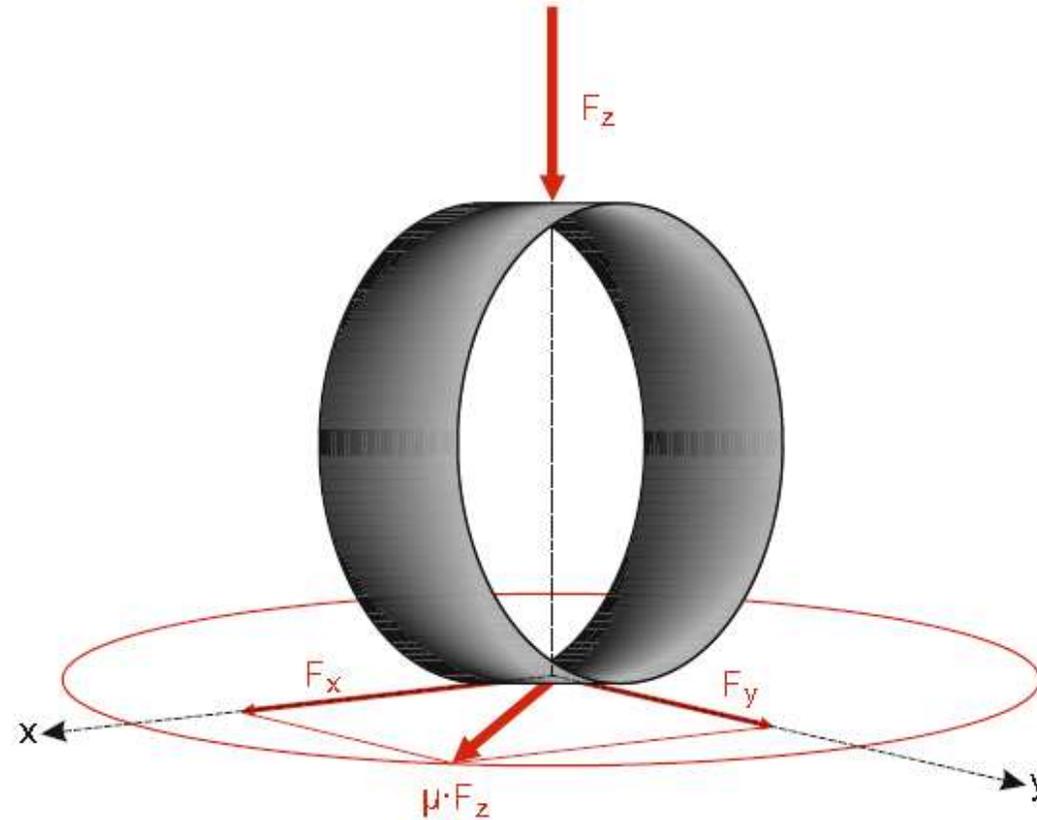
$$\mu_x = \frac{F_x}{F_z}$$

$$\mu_y = \frac{F_y}{F_z}$$

Different Friction Coefficient along Lateral and Longitudinal Axis



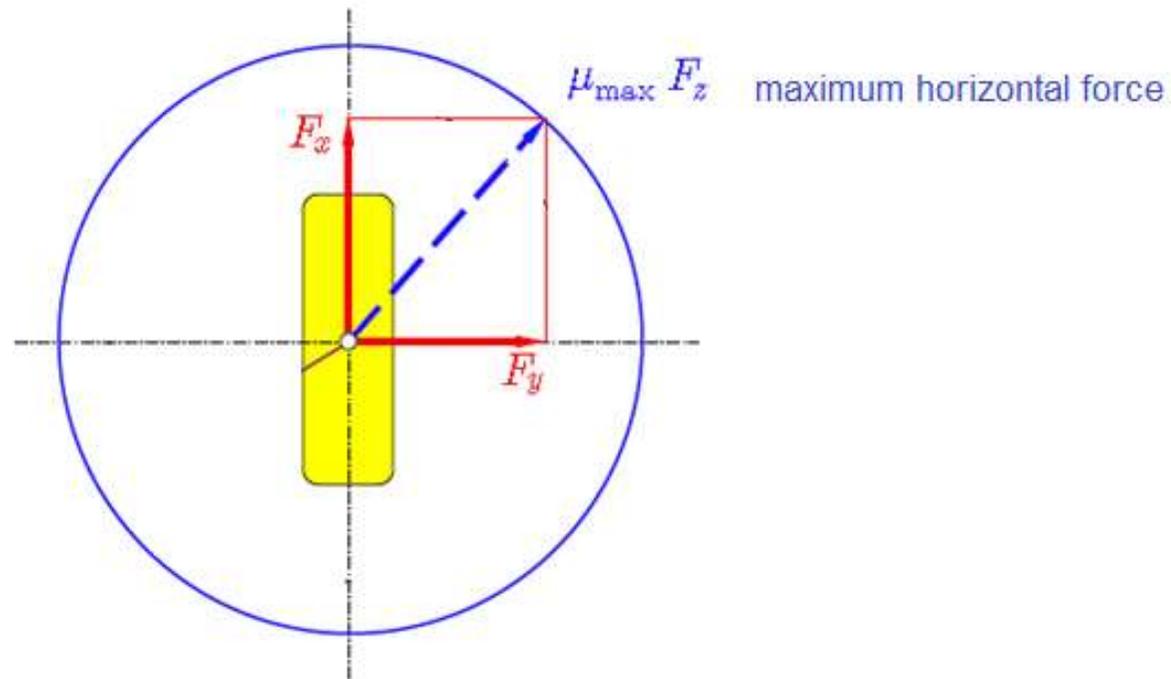
## Circle of friction



$$\mu_{max} F_z \geq \sqrt{F_x^2 + F_y^2}$$

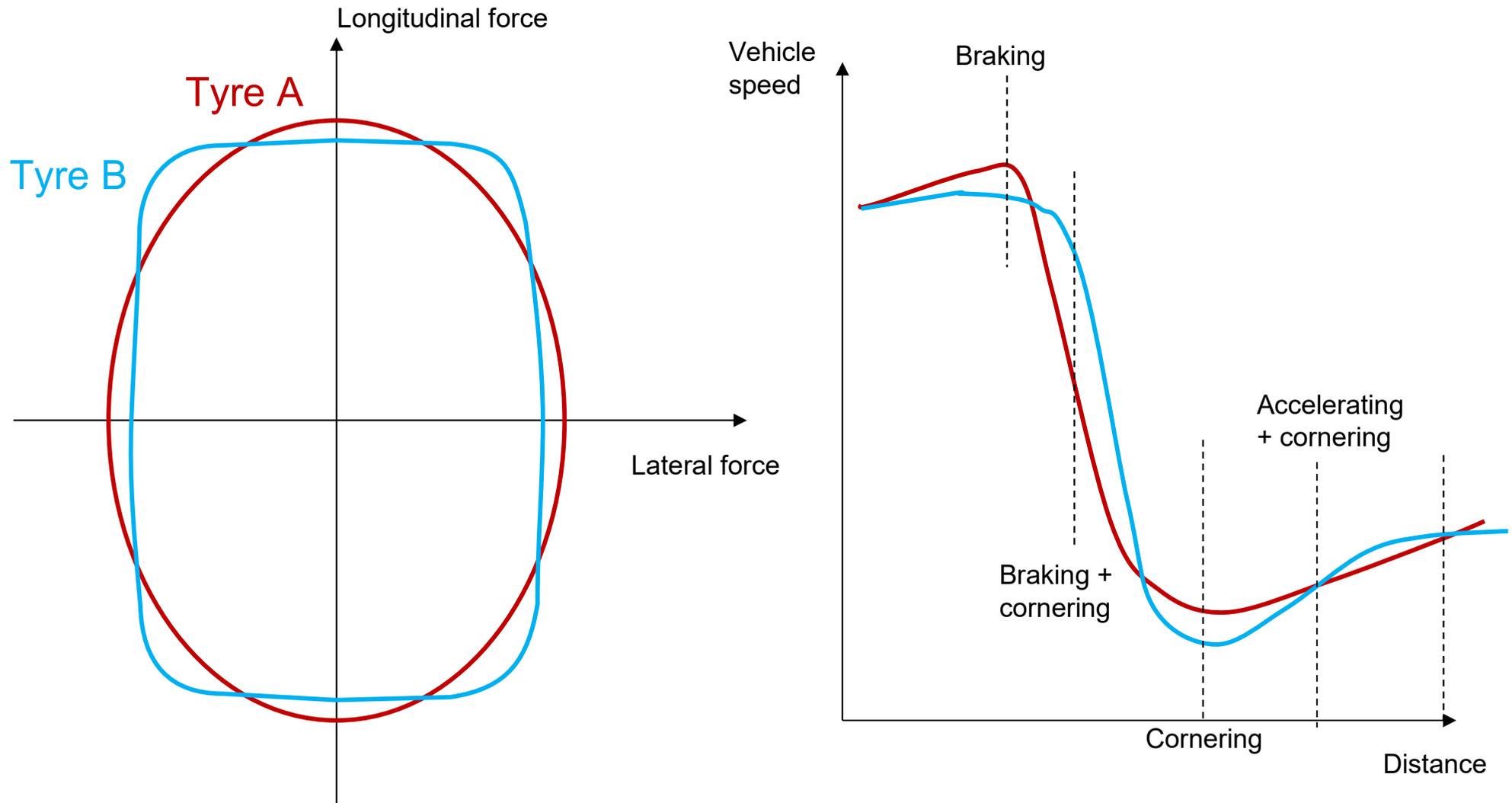
# Kamm circle

## Circle of friction



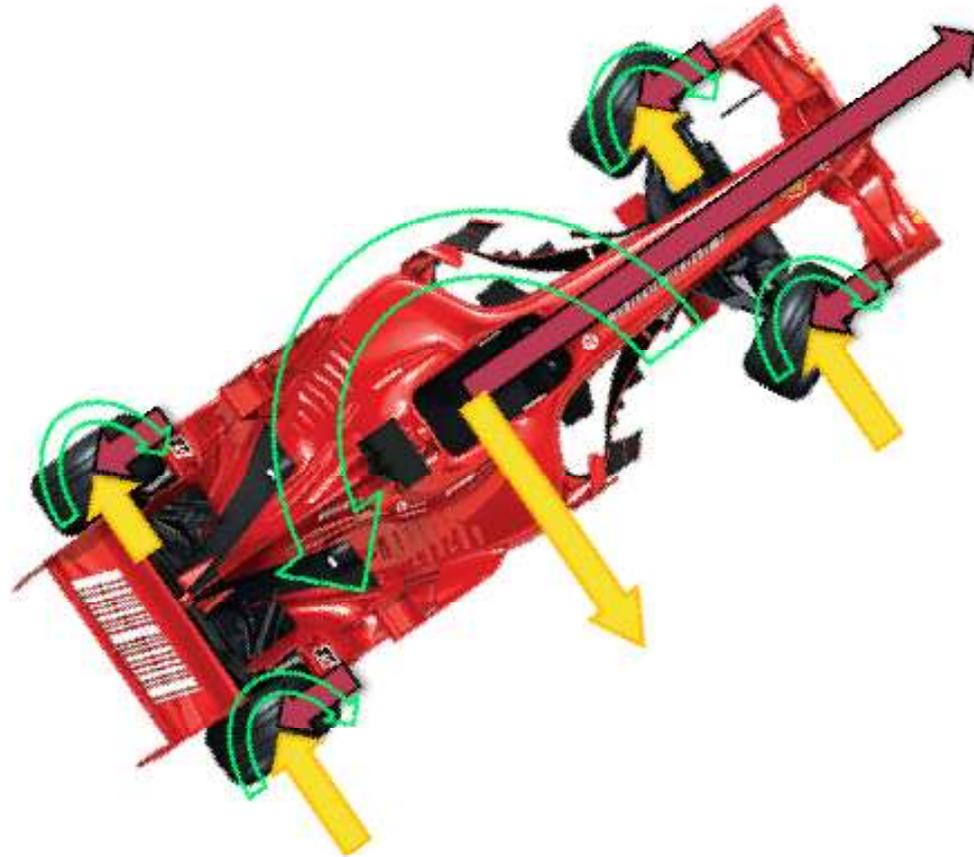
$$\sqrt{F_x^2 + F_y^2} \leq \mu_{\max} F_z$$

# Different tires



# Tyre forces

- weight of the car
- lateral G
- longitudinal G
- vertical aero
- longitudinal aero
- lateral aero



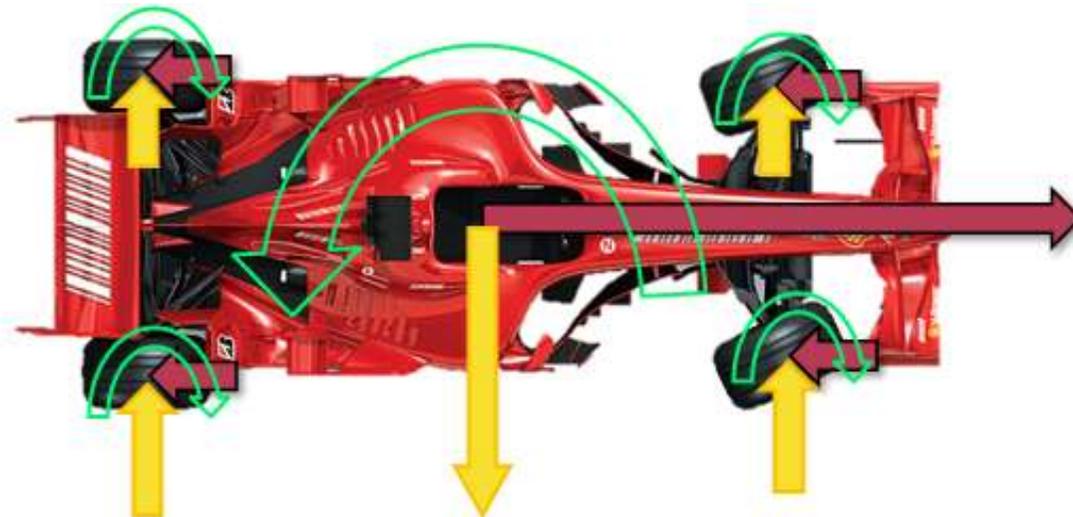
# Tyre moments

- Overturning moment  $M_x$
- Rolling resistance  $M_y$
- Aligning torque  $M_z$



# Tyre forces and moments

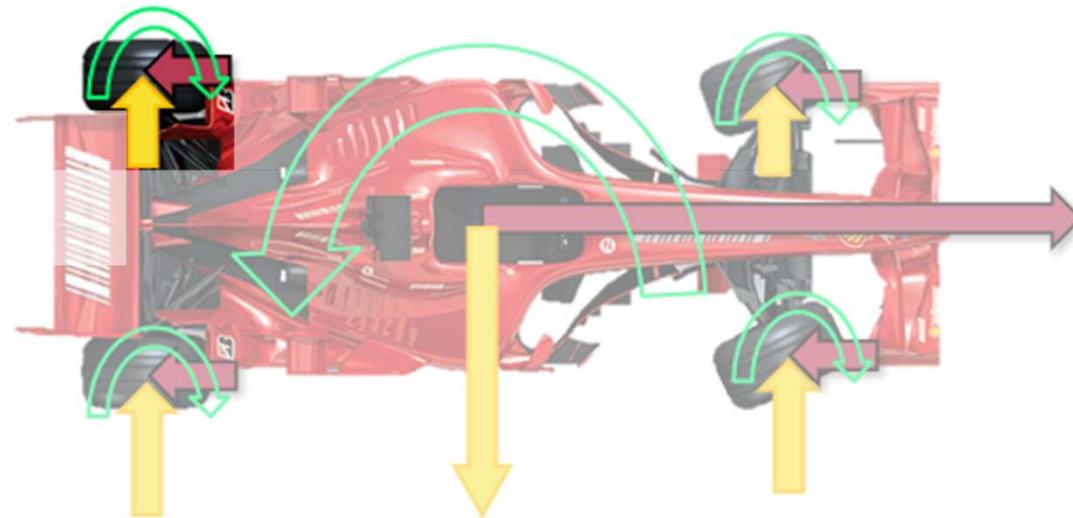
- Tyre forces and moments are function of tyre compound and structure, ground and ...



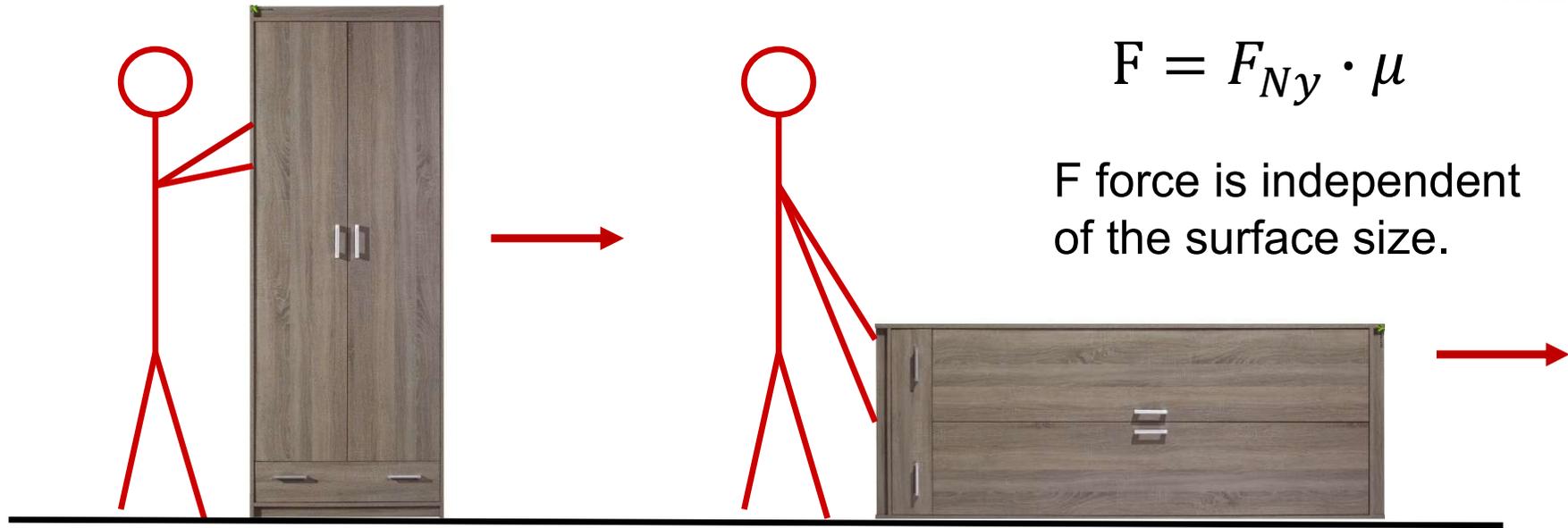
- slip angle
- slip ratio
- vertical load
- inclination angle
- speed
- pressure
- temperature
- rim design
- tyre wear...

# Source of yaw moment

- $F_x$ ,  $F_y$  and  $M_z$

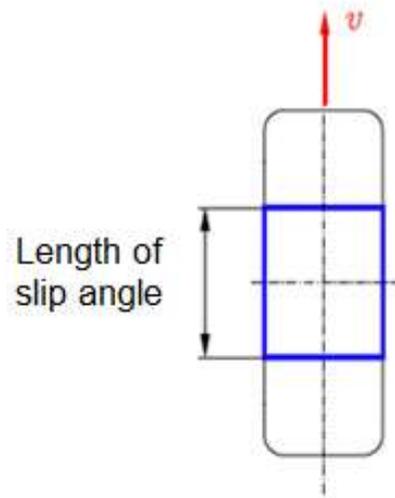


# Why slick?

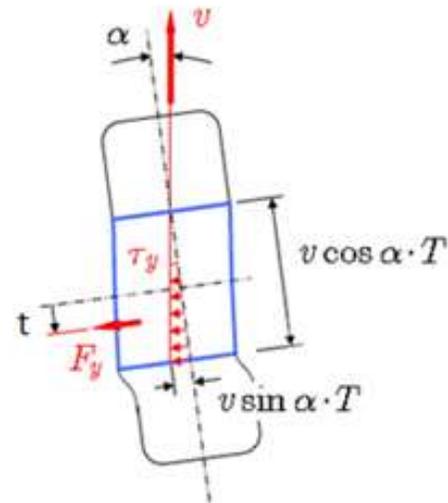


# Lateral force and cornering stiffness

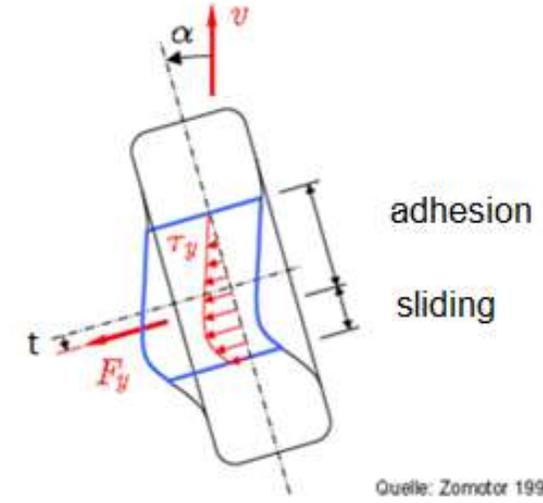
Run straight, no slip angle



Small slip angle



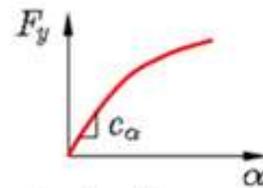
Large slip angle



Quelle: Zomotor 1991

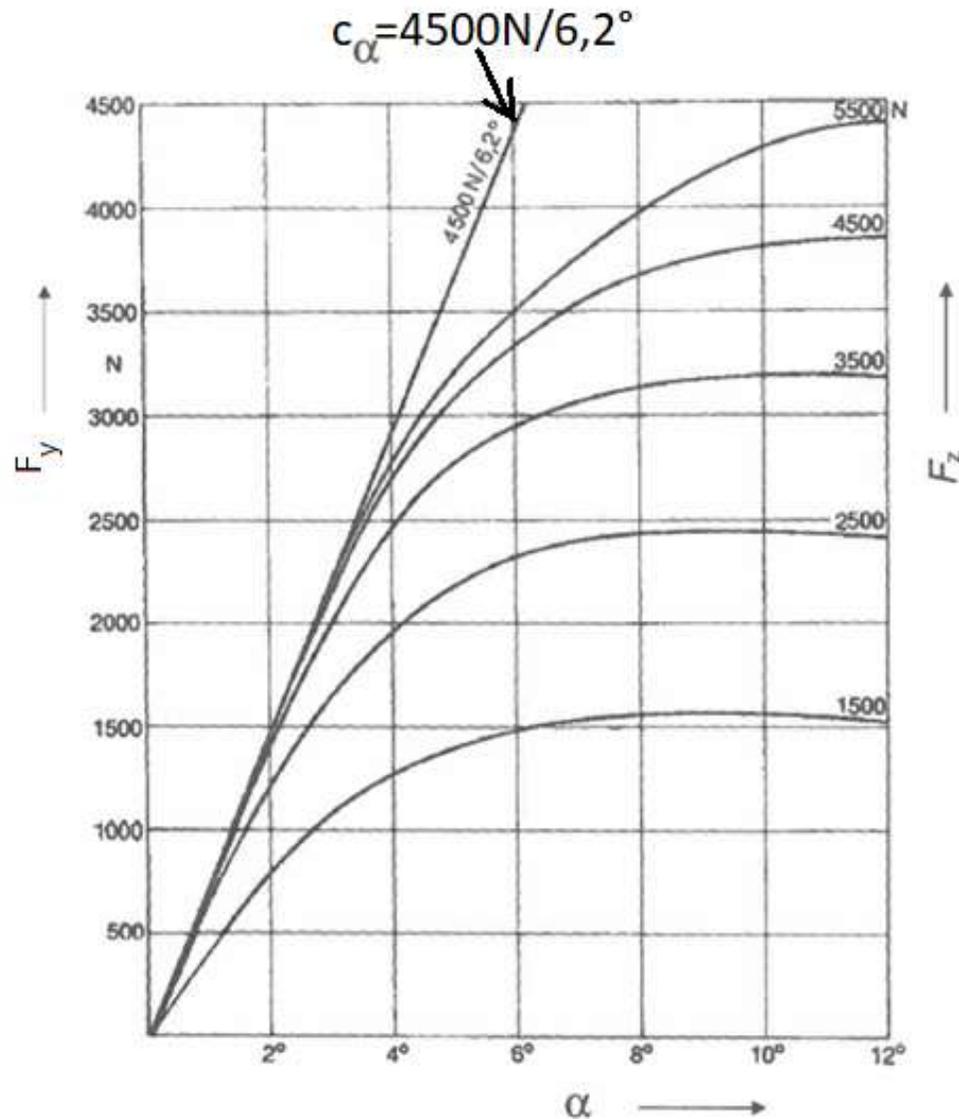
lateral force  $\sim$  area below the  $\tau_y(x)$  curve

$$F_y = c_\alpha \sin \alpha \cdot \cos \alpha \approx c_\alpha \alpha$$



cornering stiffness  $c_\alpha$  [N/rad]

# Lateral force and cornering stiffness



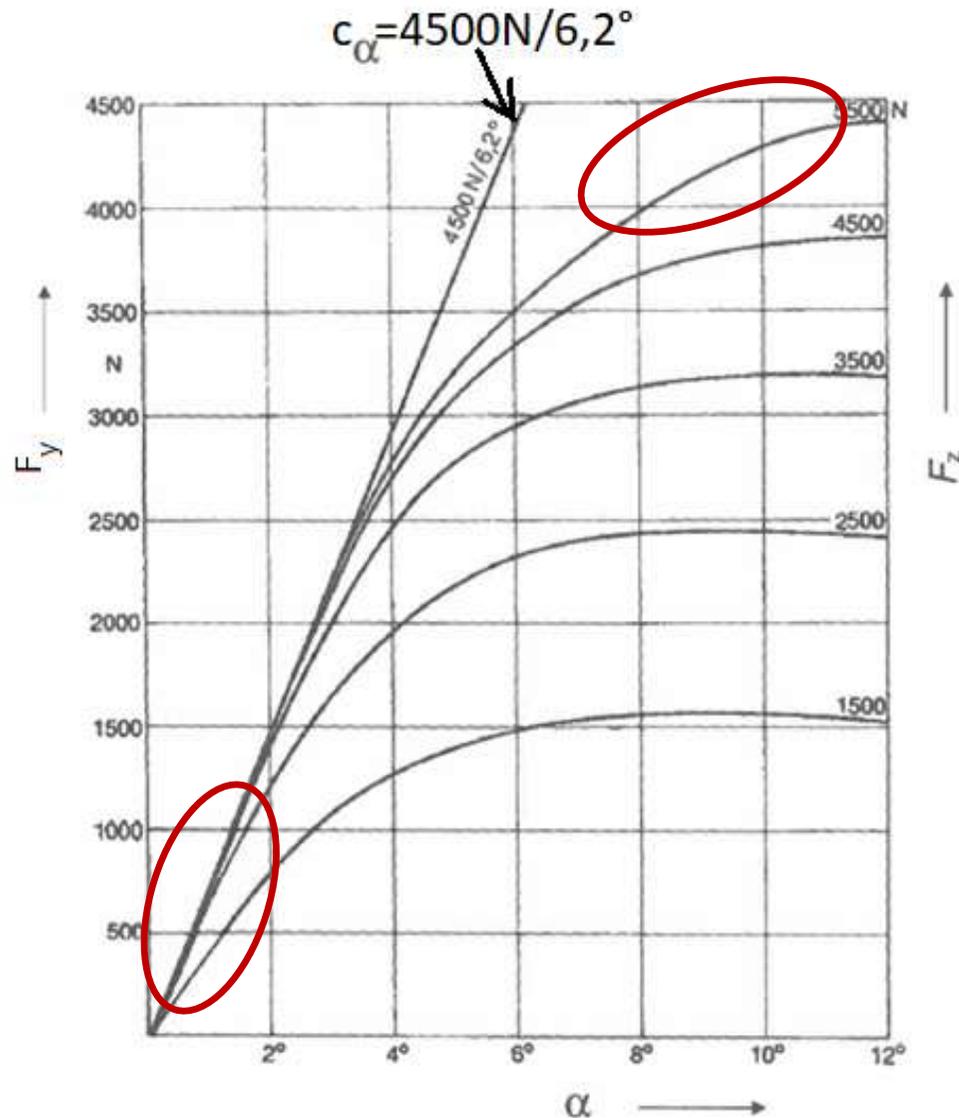
Tyre: 175 HR 14

pressure: 2.3 bar

cornering stiffness:  $C_\alpha = 41586 \text{ N/rad}$

The same tyre has different behaviour on different vehicles, therefore the value of  $C_\alpha$  will be different as well.

# Lateral force and cornering stiffness

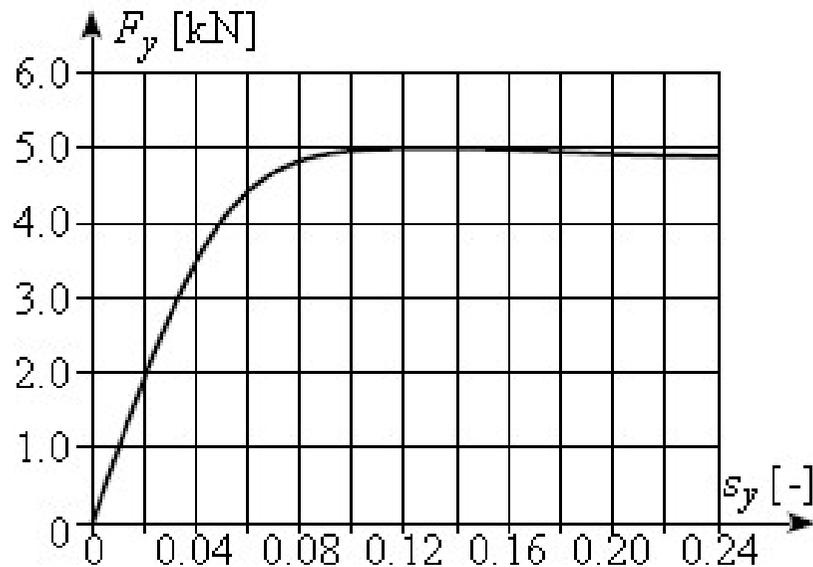


$$m \cdot a_y = F_{y,F}(\alpha) + F_{y,R}(\alpha)$$

$$\alpha = \alpha(v, \omega, \dots)$$

- calculation is easier – reason to linearize
- the value of slip angle in that range where we use tyre is low, so we are allowed to linearize

# Test example



Tyre test bench measurement data can be seen on diagram.

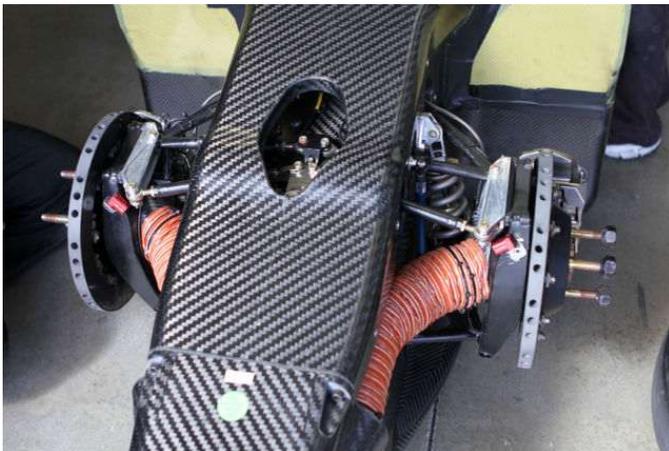
a) What value is the cornering stiffness?

b) Draw an approximately correct graph with the same tyre as it is marked in wet conditions! The nature of the curve should be correct!

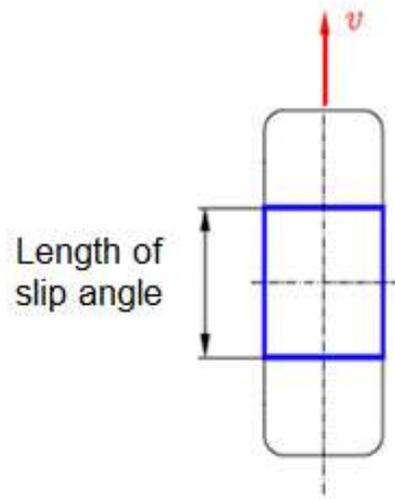
# Cornering stiffness distribution

## Nissan Deltawing racecar

- Cornering stiffness scales roughly linearly with tyre width
- Guessing WD of a particular car if you look at the relative tyre width front to rear
- Good example: Deltawing sportscar
  - 4 inch wide front tyres
  - If far enough from CoG, reasonable yaw moment if can be generated

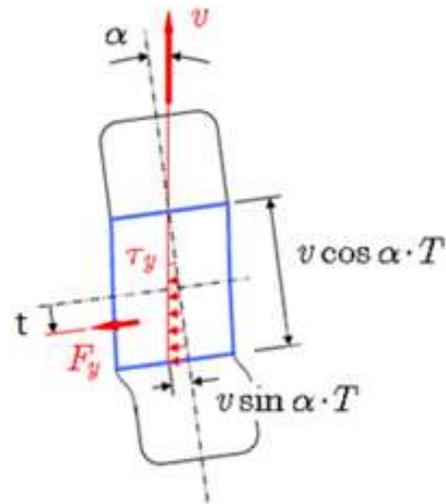


Run straight, no slip angle

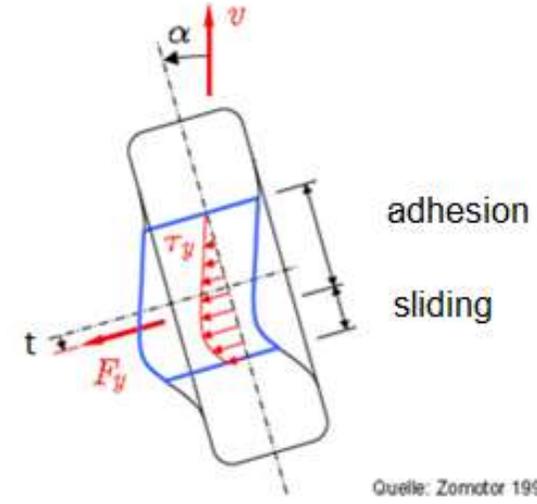


Length of  
slip angle

Small slip angle



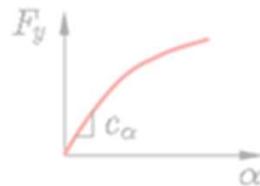
Large slip angle



Quelle: Zornator 1991

lateral force  $\sim$  area below the  $\tau_y(x)$  curve

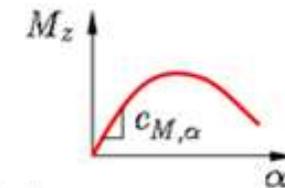
$$F_y = c_\alpha \sin \alpha \cdot \cos \alpha \approx c_\alpha \alpha$$



cornering stiffness  $c_\alpha$  [N/rad]

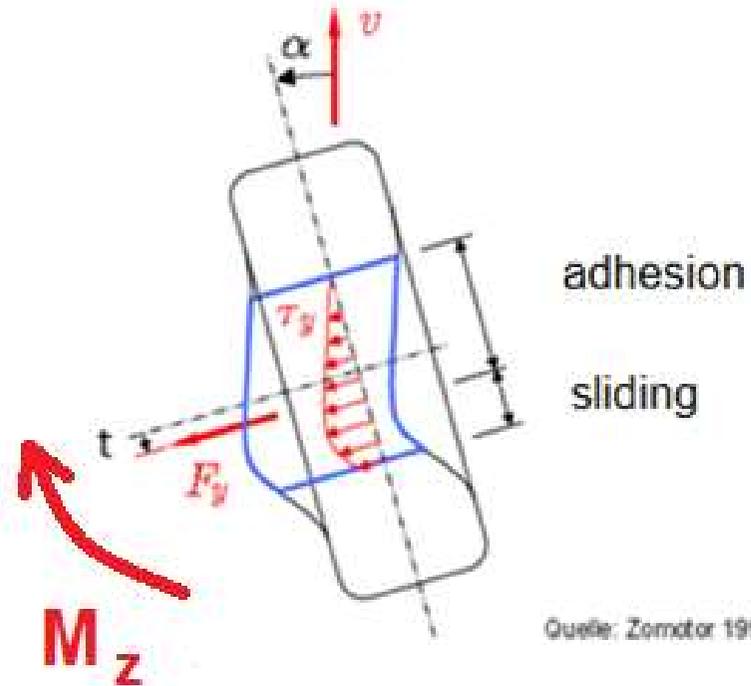
self aligning torque

$$M_z = t F_y \approx t c_\alpha \alpha$$



pneumatic trail  $t$  [m]

# Self aligning torque



- Main objective?
- Starting point - ending point?
- Complex parts?
- Less relevant part(s), could be omitted part(s)?
- Most useful part(s)?
- How could today's material have contributed to your professional goals?

- [https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.wired.com%2F2008%2F05%2Fff-formulaone%2F&psig=AOvVaw1Jpc66dH0FE0zK\\_Le6ev-D&ust=1612202433180000&source=images&cd=vfe&ved=0CAIQjRxqFwoTCNDU7Z\\_gxu4CFQAAAAAdAAAAABAD](https://www.google.com/url?sa=i&url=https%3A%2F%2Fwww.wired.com%2F2008%2F05%2Fff-formulaone%2F&psig=AOvVaw1Jpc66dH0FE0zK_Le6ev-D&ust=1612202433180000&source=images&cd=vfe&ved=0CAIQjRxqFwoTCNDU7Z_gxu4CFQAAAAAdAAAAABAD)
- <https://www.google.com/url?sa=i&url=https%3A%2F%2Fthecuriousastronomer.wordpress.com%2Ftag%2Fangular-velocity%2F&psig=AOvVaw0VWlzbG2-A6w7ca5mULVB&ust=1612208234344000&source=images&cd=vfe&ved=0CAIQjRxqFwoTCJjTkuv1xu4CFQAAAAAdAAAAABAY>
- [https://www.google.com/imgres?imgurl=https%3A%2F%2Fi0.wp.com%2Fotrwheel.com%2Fwp-content%2Fuploads%2F2016%2F08%2FradialvsbiasplyFINAL.jpg%3Fresize%3D630%252C301&imgrefurl=https%3A%2F%2Fotrwheel.com%2Fotr-blog%2Fradial-vs-bias-need-know%2F&tbnid=y0Ea\\_MrsrdG3gM&vet=12ahUKEwi4sPLjvM7uAhUjqnEKHTx0AewQMygFegUIARCiAQ..i&docid=0Oz8itZjkydoIM&w=630&h=301&q=tyre%20%20bias%20ply%20radial&ved=2ahUKEwi4sPLjvM7uAhUjqnEKHTx0AewQMygFegUIARCiAQ](https://www.google.com/imgres?imgurl=https%3A%2F%2Fi0.wp.com%2Fotrwheel.com%2Fwp-content%2Fuploads%2F2016%2F08%2FradialvsbiasplyFINAL.jpg%3Fresize%3D630%252C301&imgrefurl=https%3A%2F%2Fotrwheel.com%2Fotr-blog%2Fradial-vs-bias-need-know%2F&tbnid=y0Ea_MrsrdG3gM&vet=12ahUKEwi4sPLjvM7uAhUjqnEKHTx0AewQMygFegUIARCiAQ..i&docid=0Oz8itZjkydoIM&w=630&h=301&q=tyre%20%20bias%20ply%20radial&ved=2ahUKEwi4sPLjvM7uAhUjqnEKHTx0AewQMygFegUIARCiAQ)

**Thank you for your attention!**

