

# Fuel consumption simulation

Vehicle dynamics



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## Analytic estimation in a steady state operation point

Steady state motion equation:

$$\left. \begin{aligned} F_{tr} &= m_v g (\sin\alpha + f \cos\alpha) + \frac{1}{2} c_w A_v \rho_a v_v^2 \\ F_{tr} &= \eta \frac{M_e i_{gb} i_{fg}}{r_w} \end{aligned} \right\} \Rightarrow M_m = \frac{r_w \left[ m_v g (\sin\alpha + f \cos\alpha) + \frac{1}{2} \rho_a c_w A_v v_v^2 \right]}{\eta i_{gb} i_{fg}}$$

Actual engine speed:

$$\omega_e = v_v \frac{i_{gb} i_{fg}}{r_w}$$

Actual engine power:

$$P_e = \omega_e M_e$$

# Fuel consumption simulation

## Analytic estimation in a steady state operation point

The actual fuel mass flow rate (based on the brake specific fuel consumption characteristics):

$$\dot{m}_f = b_{spec} P_e \left[ \frac{kg}{s} \right]$$

Attention: the unit of the BSFC is usually  $\left[ \frac{g}{kWh} \right]!$

The actual fuel mass flow rate (based on the effective efficiency of the engine):

$$\dot{m}_f = \frac{P_e}{\eta_{e,eff} H_f}$$

The BSFC and the effective efficiency can be determined from each other.

The fuel consumption on 100km:

$$V_t = 10^8 \frac{\dot{m}_f}{v_v \rho_f} [l/100 \text{ km}]$$

# Fuel consumption simulation



## Analytic estimation in a steady state operation point

How much is the actual fuel consumption (related to 100km) of the vehicle that has 15 tons, it runs up on a 5% angle slope with 70km/h. The speed is constant, the engine's actual effective efficiency is 35%.

Given data: the fuel heat capacity 44.8 MJ/kg, fuel density 830 kg/m<sup>3</sup>, vehicle's front area and drag coefficient: 5 m<sup>2</sup> and 0.4, rolling resistance: 0.015, air density: 1.2 kg/m<sup>3</sup>.

# Fuel consumption simulation

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$$\alpha = \arctg(0.05) = 2.86^\circ$$

$$F_{tr} = mg(\sin\alpha + f\cos\alpha) + \frac{1}{2}c_w A\rho v^2$$

$$= 10000 \cdot 9.81 \cdot (\sin 2.86^\circ + 0.015 \cdot \cos 2.86^\circ) + \frac{1}{2} \cdot 0.4 \cdot 5 \cdot 1.2 \cdot \left(\frac{70}{3.6}\right)^2 = 6818N$$

$$P_{tr} = F_{tr}v = 6818 \cdot \frac{70}{3.6} = 132575W$$

$$V_f = \frac{10^8 P_e}{\eta_{m,eff} H_f v \rho_f} = \frac{10^8 \cdot 132575}{0.35 \cdot 44.8 \cdot 10^6 \cdot \frac{70}{3.6} \cdot 830} = 52.4 \text{ l/100km}$$

## Dynamic fuel consumption simulation

### Presumptions:

- The vehicle speed is higher than the minimum stable speed,
- The vehicle runs on flat ground,
- The time of gear changing is negligible.

### Boundary conditions:

- The input (reference signal) vehicle speed profile in function of the time

### Simplified driver model:

- Throttle control input ( $0 \leq u_{throttle} \leq 1$ )
- Brake control input ( $0 \leq u_{brake} \leq 1$ )
- Gear selection control

# Fuel consumption simulation

## Fuel consumption simulation – vehicle motion equations

$$\delta m_v \ddot{s}_v = F_{tr} - F_{brake} - m_v g f - \frac{1}{2} \rho_a c_w A_v v_v^2$$

$$F_{tr} = \frac{M_e i_{gb} i_{fg} \eta}{r_w}$$
$$\omega_e = \frac{v_v i_{gb} i_{fg}}{r_w}$$

$M_e = u_{throttle} M_{e,max}(\omega_e)$  - maximum torque characteristic is given

$F_b = u_{brake} F_{brake,max}$  - maximum brake force is also given

$v_v \leq v_{limit}$  - speed limit

$\dot{m}_f = b_{spec} P_e$  - BSFC characteristic is given

$$V_f = \frac{\dot{m}_f}{\rho_f v_v}$$

## Fuel consumption simulation – driver modelling

Actual gear selection: the selected gear should provide the the smallest difference from the pre-given target engine speed:

$$i_{gb,aim} = \frac{\omega_{e,aim} r_w}{v_v i_{fg}} \Rightarrow i_{gb} = i_{gb,i} \Big|_{|i_{gb,aim} - i_{gb,i}| = \min!, i=1,2,\dots,n}$$

The state change of the throttle control is proportional with difference between the actual and the target vehicle speed (with a tolerance interval):

$$\frac{du_{throttle}}{dt} = \begin{cases} -P_{throttle}(v_v - v_{ref} + v_{tol}), & \text{if } v_v - v_{ref} < -v_{tol} \\ 0, & \text{if } |v_v - v_{ref}| < v_{tol} \\ -P_{throttle}(v_v - v_{ref} - v_{tol}), & \text{if } v_v - v_{ref} > v_{tol} \end{cases}, \text{ but } 0 \leq u_{throttle} \leq 1$$

The state change of the brake control is proportional with difference between the actual and the target vehicle speed (with a tolerance interval). There is no throttling and braking at the same time:

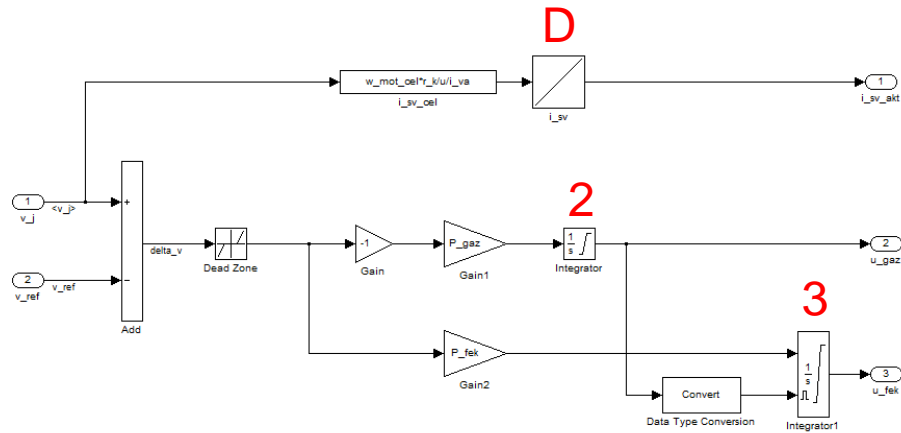
$$\frac{du_{brake}}{dt} = \begin{cases} P_{brake}(v_v - v_{ref} + v_{tol}), & \text{if } v_v - v_{ref} < -v_{tol} \\ 0, & \text{if } |v_v - v_{ref}| < v_{tol} \\ P_{brake}(v_v - v_{ref} - v_{tol}), & \text{if } v_v - v_{ref} > v_{tol} \end{cases}, \text{ but } 0 \leq u_{brake} \leq 1 \text{ and } u_{brake} = 0, \text{ if } u_{throttle} > 0$$





# Fuel consumption simulation

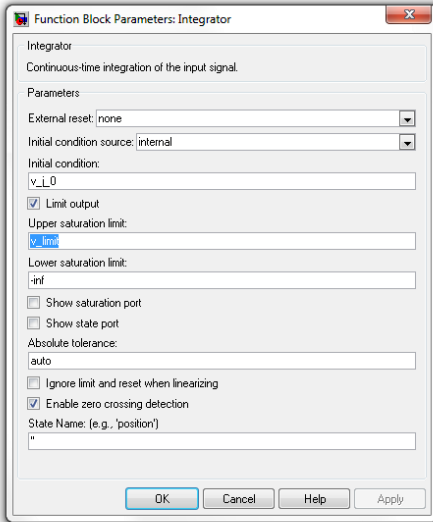
## Dynamic fuel consumption simulation



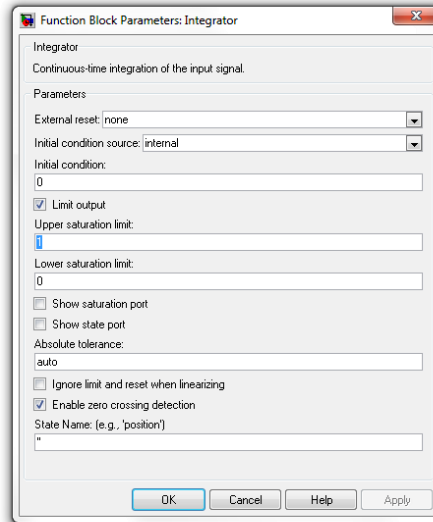
# Fuel consumption simulation

## Dynamic fuel consumption simulation

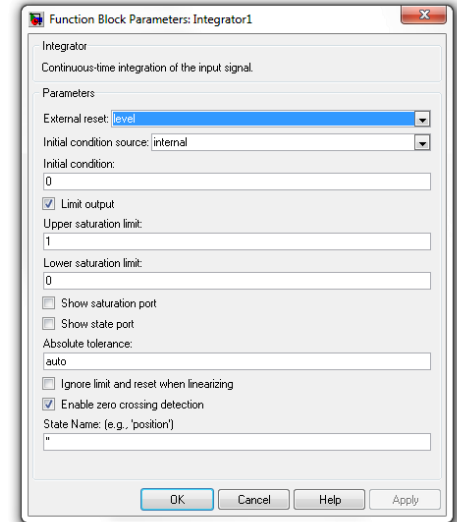
1



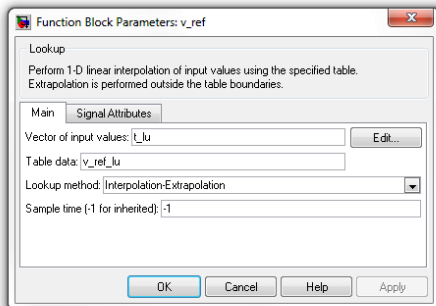
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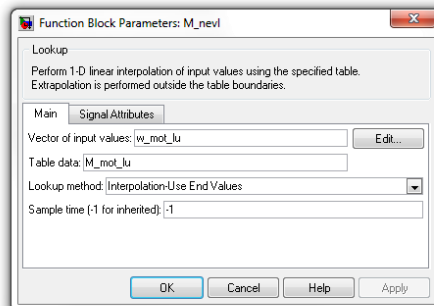
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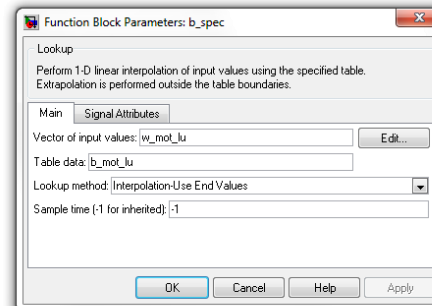
A



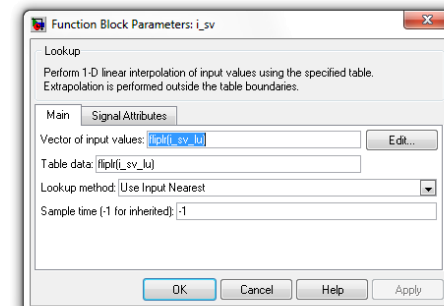
B



C



D



# Fuel consumption simulation

## Dynamic fuel consumption simulation

$m_j=15000$ ; % [kg] vehicle mass  
 $\delta=1.1$ ; % [-] rolling mass factor  
 $A_j=5$ ; % [m<sup>2</sup>] front area  
 $c_w=0.5$ ; % [-] drag coefficient

$i_{va}=3.5$ ; % [-] final gear ratio  
 $r_k=0.4$ ; % [m] wheel rolling radius  
 $\eta=0.95$ ; % [-] mechanical efficiency of the powertrain

$F_{fek\_max}=100000$ ; % [N] max brake force

$f=0.015$ ; % [-] rolling resistance factor  
 $\rho_l=1.293$ ; % [kg/m<sup>3</sup>] air density  
 $g=9.81$ ; % [m/s<sup>2</sup>] gravitational acceleration  
 $\rho_t=830$ ; % [kg/m<sup>3</sup>] fuel density

$w_{mot\_cel}=1500 \cdot 2 \cdot \pi / 60$ ; % [rad/s] target engine speed  
 $v_{limit}=90/3.6$ ; % [m/s] speed limit  
 $v_{tol}=1/3.6$ ; % [m/s] speed tolerance interval

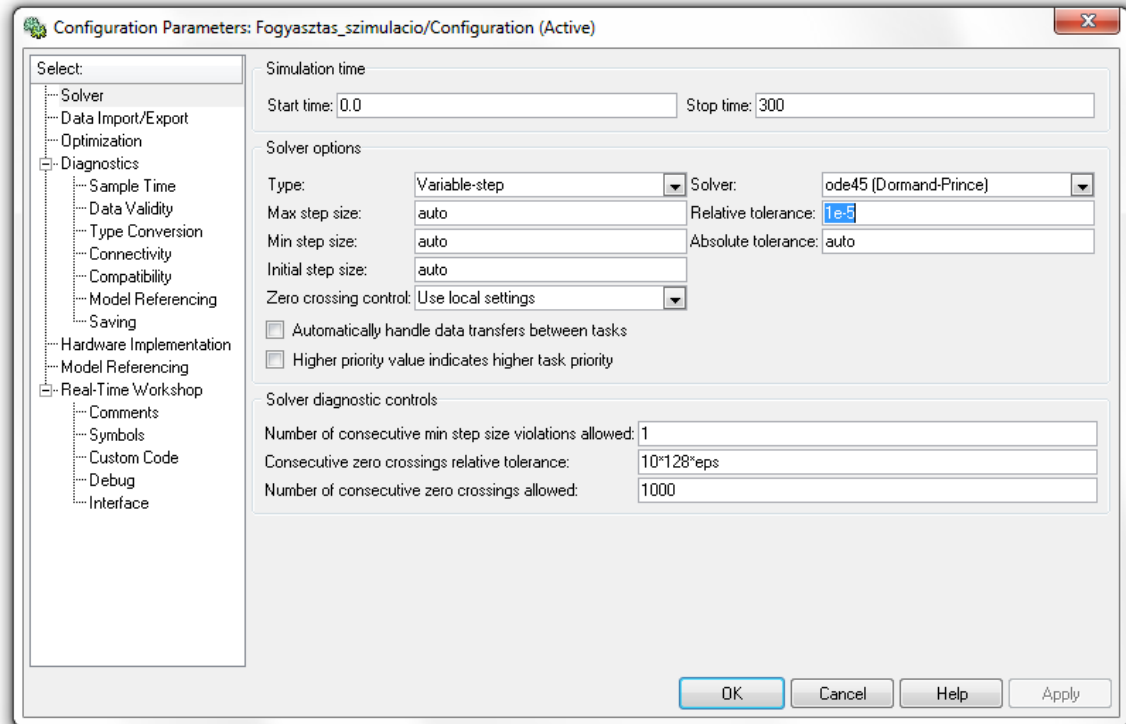
% Engine characteristics  
 $w_{mot\_lu}=[0 \ 800 \ 1000 \ 1500 \ 2000 \ 2500 \ 2600] \cdot 2 \cdot \pi / 60$ ; % [rad/s]  
 $M_{mot\_lu}=[0 \ 560 \ 650 \ 660 \ 600 \ 500 \ 0]$ ; % [Nm]  
 $b_{mot\_lu}=[500 \ 230 \ 215 \ 200 \ 210 \ 220 \ 500]$ ; % [g/kWh]

% Gear ratios  
 $i_{sv\_lu}=[6.3 \ 3.5 \ 2.1 \ 1.4 \ 1 \ 0.8]$ ; % [-]

% Driver model  
 $P_{gaz}=0.2$ ; % [1/m] throttle controller proportional part  
 $P_{fek}=0.05$ ; % [1/m] brake controller proportional part

% Reference speed profile  
 $t_{lu}=[0 \ 20 \ 40 \ 80 \ 200 \ 220 \ 250 \ 300]$ ; % [s]  
 $v_{ref\_lu}=[10 \ 50 \ 50 \ 70 \ 70 \ 50 \ 50 \ 10]/3.6$ ; % [m/s]

% Initial conditions  
 $v_j_0=1000 \cdot 2 \cdot \pi / 60 \cdot r_k / i_{va} / i_{sv\_lu}(1)$ ; % [rad/s] initial vehicle speed



# Thank you for your attention!

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